JOINT INSTITUTE FOR NUCLEAR RESEARCH
Veksler and Baldin Laboratory of High Energy Physics

FINAL REPORT ON THE START PROGRAMME

Computer simulations of betatron matching beam line into Nuclotron

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Оглавление

Abstract

This work is aimed for matching beam line in Nuclotron using the response matrix, and using the matrix singular decomposition algorithm. A program was written to match the Booster-Nuclotron channel.

1. Introduction

The NICA (Nuclotron based Ion Collider fAcility) project aims to create an accelerator complex for conducting research in the field of particle physics at JINR (Dubna, Russia) in a previously inaccessible area of experimental parameters and condition. Accelerator complex plays a pivotal role in contemporary research within the realms of elementary particle physics and nuclear physics.

The life of a beam begins in the ion source, where particles are born and, by means of electric fields, are “pulled” into the pre-accelerator. In it, the beam is formed and transported to the linear accelerator, where the particles acquire the required energy and move through the beam transport channels to the Booster. The Booster is a superconducting synchrotron, which allows for acceleration and not for the loss of particles during the acceleration process before they get to the Nuclotron, since the main losses occur at low energies. The Nuclotron accelerates ions from the Booster to energies sufficient for conducting an experiment in the Collider. The Collider, which is very similar in structure to the Booster and the Nuclotron, has two rings converging at two meeting points. This allows the beams to move towards each other for a long time and interact with each other only where the apertures of the rings converge.

The mismatch between the channel and the accelerator leads to an increase in longitudinal and transverse emittance. The increase of emittance due to betatron mismatch has negative effect on beam transmission increasing particles losses at first turns in the accelerator.

In this work the main object is the Booster-Nuclotron channel of the NICA complex.

The purpose of this work is to develop methods, algorithms, and software for matching beam line into Nuclotron. The key tasks of the work are as follows:

1. Develop an algorithm for correcting the beta function at the channel output using quadrupoles based on the matrix inversion method using singular value decomposition of matrices;
2. Create a program for controlling the Booster-Nuclotron channel matching with a user interface.

1.1. The NICA accelerator complex

In 2009, work began at JINR on the design and construction of a new accelerator complex, NICA (Nuclotron-based Ion Collider fAcility) [1].

The goal of the NICA/MPD (Nuclotron based Ion Collider fAcility and Multi Purpose Detector) project is to create an accelerator complex designed to carry out an advanced particle physics research program at JINR. Experiments will be carried out on fixed targets using Nuclotron beams at kinetic energy up to the maximum design energy (4.5 GeV/n). A Multi-purpose Detector (MPD) has been proposed for the Collider experiment. Another goal of the NICA project is to conduct experimental research on spin physics on oncoming polarized beams of protons and light nuclei[2].

The new NICA accelerator complex will provide beams of various particles with a wide range of parameters. This will allow for both applied and fundamental research in various fields of science and technology. Among them are the following[2]:

- Radiobiology and space medicine;
- Cancer therapy;
- Development of accelerator beam-controlled reactors ("energy production" with subcritical assembly) and technologies for the transmutation of nuclear energy waste;
- Testing the radiation resistance of electronic devices.

The most important fundamental research directions in this field include:

1. Nature and Properties of Strong Interactions: Studying the strong interactions between the elementary constituents of the Standard Model of particle physics, namely quarks and gluons.
2. Phase Transition Search: Searching for signs of a phase transition between hadronic matter and QGP (Quark-Gluon Plasma), as well as exploring new states of baryonic matter.
3. Study of Strong Interaction and QGP Symmetry: Investigating the fundamental properties of strong interactions and QGP symmetry [3].

Applied research using particle beams produced at the NICA complex is aimed at developing new technologies in materials science, solving environmental problems, creating new methods of energy production, cancer therapy, etc.

The main accelerator installations of the NICA complex are an injection complex, a Booster, an upgraded Nuclotron and two storage rings with two beam meeting points.

Figure 1 - Schematic view of the NICA accelerator complex[4]

1.2. Booster design and systems

The main tasks of the booster are as follows[2]:

1. accumulation of ions at an injection energy of 2.5 * 10^9 $^{197}Au^{31+}$ ions;
2. effective acceleration of incompletely stripped ions, which is possible due to the ultrahigh vacuum in the beam chamber;
3. acceleration of heavy ions to the energy required for their efficient reduction of internal energy;
4. fast (single-lap) extraction of the accelerated beam for its injection into the Nuclotron.

A booster with a perimeter of 211 m and a structure of four periods is placed inside the yoke of a Synchro-phasotron magnet. The duration of the Booster's operating cycle is 4.02 seconds. If necessary, a technological pause between cycles of 1 second duration is possible.

Accumulation of gold ions and their acceleration to an energy of 578 MeV/ n, which is sufficient for subsequent stripping them to the state $^{197}Au^{79+}$. This makes it possible to significantly reduce the pressure requirements for the residual gas in the Nuclotron due to a decrease in the probability of ion recharge. At the same time, the final energy of the gold nuclei in the Nuclotron is 4.5 GeV/n. The use of electronic cooling in a Booster with an ion energy of 65 MeV/n will lead to a decrease in the longitudinal emittance of the beam to the value required for compression of the clot upon completion of its acceleration in the Booster.

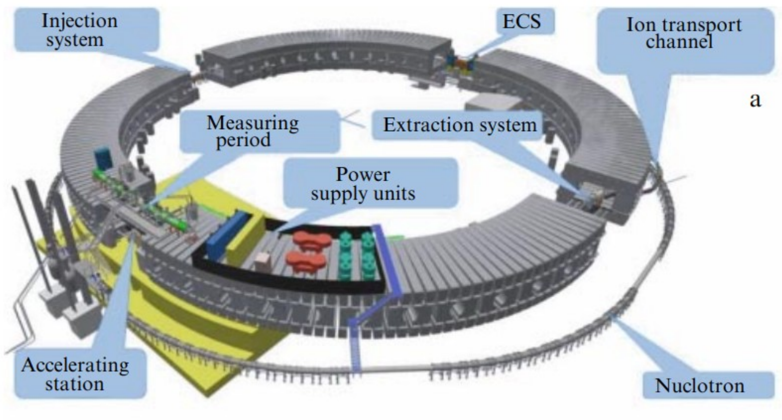

Figure 2 - Placement of the main booster systems (conventionally numbered along the beam path (counterclockwise) starting from the injection site). Straight gaps are designed to accommodate beam injection and extraction systems, accelerating HF stations, and the ECS

1.3. Booster-Nuclotron beam transport channel

The Booster-Nuclotron beam transport line (Fig. 3) passes through the magnet yoke of the former Synchrophasotron accelerator, then the channel route descends down to The Nuclotron ring through an opening in the concrete floor above The Nuclotron tunnel. The beam line has a complex three-dimensional geometry, and beam transportation in it is performed horizontally and vertically at the same time. The transport line consists of the main path of ion transfer to the Nuclotron and a branch for the dump of the ions in non-target charge state. The optical system of the line consists of 5 dipole magnets, 8 quadrupole lenses, a septum magnet for the dump of ions of non-target charge state and 3 two coordinate dipole correctors. The total length of the line is 25.5 m. The Azimuthal size of the channel is approximately 45°, which corresponds to the injection of a beam through one superperiod (octant) of the Nuclotron from the extraction point of the Booster.

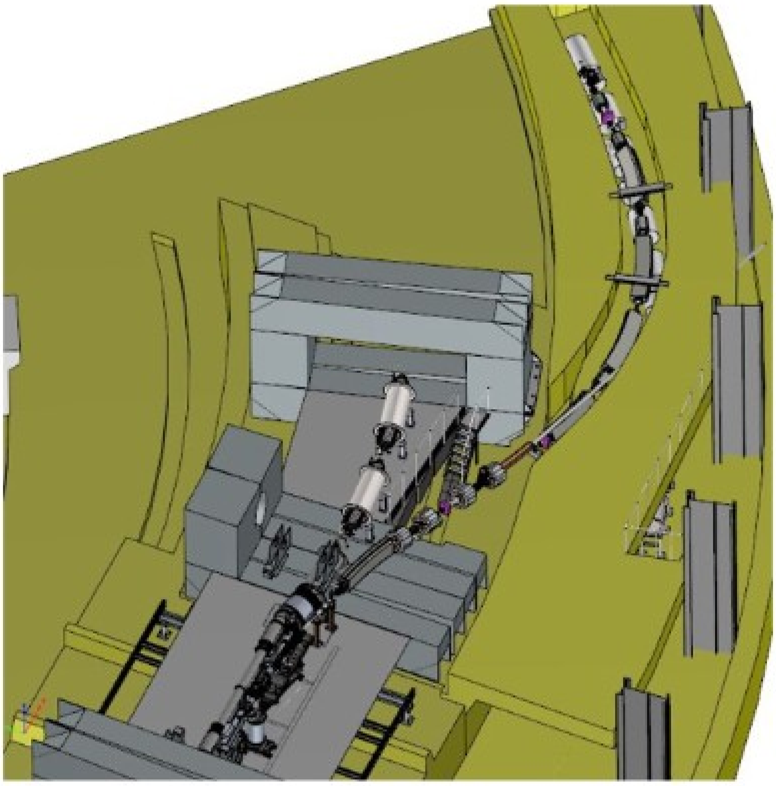

Figure 3 - View of the region containing the fast beam extraction from the Booster and Booster-Nuclotron transport line

The vacuum system of the Booster-Nuclotron transport line ensures the achievement of an operating vacuum of the order of 10-9 Torr along the ion beam pipe of the line, except for the initial section of the line with differential pumping, in which the residual gas pressure is reached at the vacuum level of the Booster ring of 10-11 Torr.

1.4. Betatron function

The motion of each ion is subject to the laws of motion of a charged particle in a superposition of electromagnetic fields of the accelerator and is described by the Newton-Lorentz equation. In the absence of electric fields, the particle is subject to the Lorentz force, which, when considering horizontal motion, is equal to:

$$F(x) = ma = q\upsilon B(x) \quad (1)$$

,

Here q , υ and a are the charge, velocity and acceleration of the particle, and the magnitude of the magnetic field depends on the horizontal coordinate of the ion, since the system contains various magnets whose fields depend on the coordinates.

To derive the equations of motion of a particle relative to the equilibrium orbit, a coordinate system is introduced that is tied to the equilibrium orbit and is called the co-moving or Frenet-Serret coordinate system (Fig. 5).

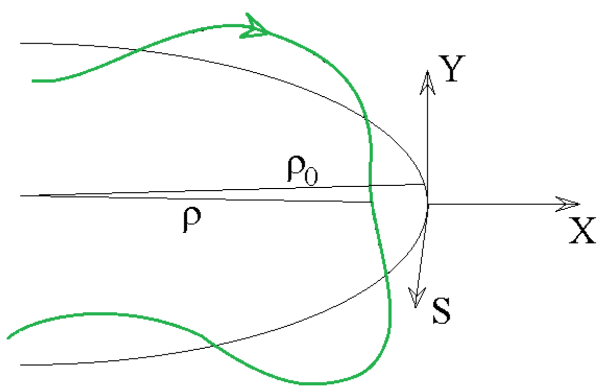

Figure 5 - The co-moving coordinate system is tied to the equilibrium orbit with radius ρ0. The coordinates of the beam particles are measured in the co-moving coordinate system.

In the co-moving coordinate system, which is curvilinear, the acceleration is the sum of the tangential and normal components:

and the equation of motion takes the form:

$$m\frac{d^2\rho}{dt^2} - m\frac{\upsilon^2}{\rho} = q\upsilon B(x) \quad (2)$$

,

In the co-moving coordinate system, the transformation is performed:

$$\frac{d}{dt} = \frac{ds}{dt}\frac{d}{ds} = \upsilon\frac{d}{ds} \quad (3)$$

,

and the radius of curvature of the ion trajectory is expressed through the radius of curvature of the equilibrium orbit as:

$$\rho = \rho_0 + x \quad (4)$$

Substituting (3) and (4) into (2), we obtain:

$$\frac{d^2x}{ds^2} - \frac{1}{\rho_0 + x} = \frac{qB(x)}{m\upsilon} \quad (5)$$

The magnetic field can be expanded in a Taylor series:

$$B(x) = B_0 + \frac{\partial B}{\partial x}x + \frac{1}{2!}\frac{\partial^2 B}{\partial x^2}x^2 + \frac{1}{3!}\frac{\partial^3 B}{\partial x^3}x^3 + \ldots \quad (6)$$

,

If in (6) we keep only linear terms, then (6) is transformed into:

$$B(x) \approx B_0 + B\rho k1x \quad (7)$$

,

Considering that $qB/m\upsilon = 1/\rho$ and the additive x is much smaller than the radius of curvature of the orbit, which allows us to use the ratio $1/(\rho_0 + x) \approx 1/\rho_0 - x/\rho_0^2$, then equation (5) is transformed into:

$$\frac{d^2x}{ds^2} + x\left(\frac{1}{\rho^2} + k1\right) = 0 \quad (8)$$

The first term in brackets (8) is responsible for weak focusing when passing through dipole magnetic fields, the second – for hard focusing by quadrupole lenses.

For vertical motion, all the above considerations are applicable, taking into account that the bending dipole magnets do not act on the vertical orbit, and the quadrupole lenses focus with the opposite sign. Therefore, the equation of motion in the vertical plane looks like:

$$\frac{d^2y}{ds^2} - k1y = 0 \qquad (9)$$

By introducing variables $K_x = \left(1/\rho^2 + k1 \right)$ and $K_y = -k1$ equations (8) and (9) they are reduced to a generalized form:

$$x'' + K_x(s)x = 0$$
$$y'' + K_y(s)y = 0 \qquad (10)$$

Equations (10) are basic in accelerator physics and are called *Hill equations*. Since the equations were derived using conventions about small deviations of the ion from the equilibrium orbit, equations (3.10) are linearized equations of small oscillations.

The solution to the Hill equation is quasi-harmonic:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta}\cos\left(\mu(s) + \mu_0 \right) \qquad (11)$$

Here ε is the beam emittance, β is the beta function, $\sqrt{\beta}$ is the Floquet function, which describes the envelope of transverse beam oscillations in the accelerator. - betatron phase advance from zero to S , μ_0 - initial phase. The difference between the betatron phase shift and the actual betatron phase value is that the phase shift in the section between two positions in the ring is calculated as the phase difference between the end and the beginning of the section.

The particle motion at each turn is a cosine curve with changing amplitude and phase (Fig. 7). Since the ion phases in the beam are different, their trajectories at each azimuth of the ring are different from each other. But they all fit into the general envelope of the beam $\sqrt{\varepsilon}\sqrt{\beta(s)}$ (Fig. 8)

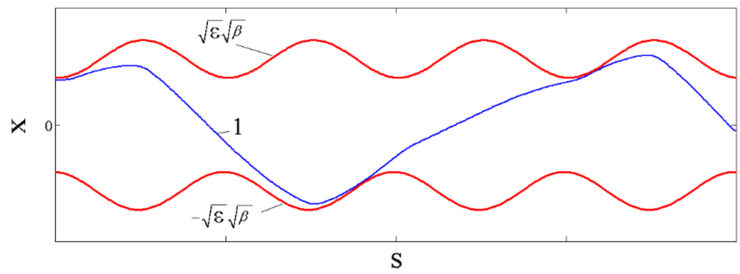

Figure 7 - Horizontal deviation of a particle from the equilibrium orbit (1) depending on the longitudinal coordinate and the beam envelope.

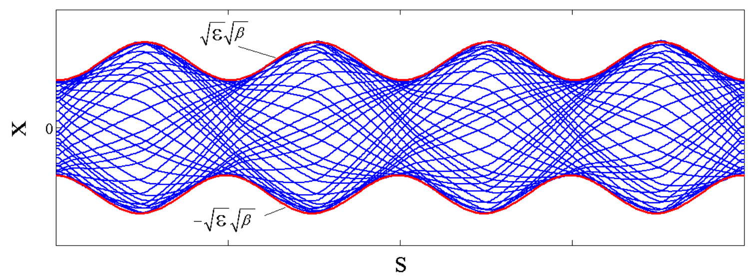

Figure 8 - Horizontal trajectory of a particle after several revolutions.

The use of beta functions to describe the transverse optics of a ring accelerator is convenient because their value does not depend on the initial distribution of ions in the beam. Beta functions are the same for any beam circulating in the ring. The dimensions of the transverse envelope are different, scaling by the square root of the emittance.

In addition to the beta function and the betatron phase, the beam envelope is also characterized by a change in the beta function, which can be expressed through the derivative:

$$\alpha(s) = -\frac{1}{2}\beta'(s)$$

The alpha parameter and beta function are equivalent to the Twiss parameters of the beam ellipse. The parameter γ is also expressed as:

The most important characteristic of transverse oscillations of particles is the frequency of betatron oscillations (betatron tune), which is equal to the number of betatron oscillations of a particle during the time it takes to complete one full revolution in the accelerator (phase shift per revolution, divided by 2π) and, accordingly, is numerically expressed as:

$$Q = \frac{\mu(L_{turn})}{2\pi} = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)}$$

1.5. Response matrix

The system response is a measurement of the Twiss parameters to the switching on of one of the quadrupole magnets. The response is calculated for each of the eight quadrupoles being switched on in turn. Then a matrix is formed from these responses, which is called the response matrix. The matrix has a dimension of 4 by 8, since 4 Twiss parameters (namely $[\beta_x, \beta_y, \alpha_x, \alpha_y]$) are being studied and there are 8 quadrupole lenses.

2. The practical part

2.1 Mathematical description of the method

Let the response matrix of size 4 by 8 be already defined.

Next, we calculate the inverse matrix using the singular value decomposition algorithm.

$$A_{m*n} = U_{m*m} \times \Lambda_{m*n} \times V^{T}_{n*n},$$

$$A^{-1} = V \times \Lambda^{-1} \times U^{T},$$

where the columns of the matrices U and V are called the left and right singular vectors, respectively, and the values of the diagonal matrix Λ are called singular numbers.

The diagonal elements of the matrix Λ have the form:

$$\lambda_1 \geq \lambda_2 \geq \lambda_3 \ldots \geq \lambda_r > \lambda_{r+1} = \ldots = \lambda_n = 0$$

where r — matrix rank of A.

If A is non-degenerate, then

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n > 0$$

And we multiply the resulting inverse matrix by the Twiss vector of parameters:

$$\vec{C} = ResponceMatrix^{-1} \times \vec{X}, \qquad (12)$$

where $\vec{C}$ - vector of gradients of fields of quadrupole lenses, and $\vec{X} = [\beta_x, \beta_y, \alpha_x, \alpha_y]$ vector of four Twiss parameters.

2.2 Calculation of the beam response matrix in the channel

The response matrix is obtained using the “OptiMX” software package, consisting of a single change in the channel quadrupole gradients.

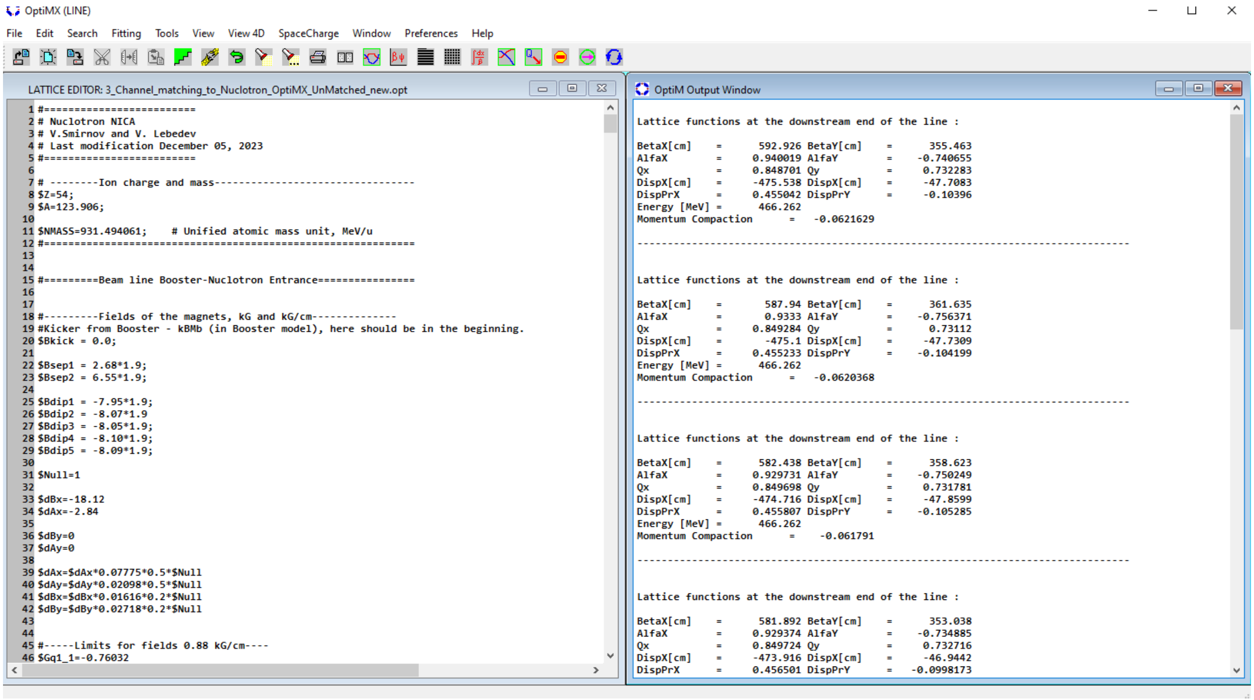

Figure 9 - Screenshot of OptiMX program

2.3 Algorithm for correcting the beta function at the channel output

At the input there are values by which the current beam parameters at the channel output need to be changed:

1. β_x-horizontal beta function
2. α_x -horizontal alpha function
3. β_y-vertical beta function
4. α_y -vertical alpha function

As a result, we obtain the input vector X:

$$\vec{X} = [\beta_x, \beta_y, \alpha_x, \alpha_y]$$

For the alpha parameter, the correction algorithm is simple, where the input vector will look like this $[\alpha_x, \alpha_y]$:

```
dG = 0.088
dAx = -αx dG
dAy = -αy dG
corrPower1 = np.zeros(shape=(8))
corrPower1[4] = 1 * dAx + 1 * dAy
corrPower1[5] = -0.674 * dAy
corrPower1[6] = 1.532 * dAx - 1.254 * dAy
corrPower1[7] = -2.112 * dAx + 1.133 * dAy
```

The result will look like this:

$$corrPower\,1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 * dAx + 1 * dAy \\ -0.674 * dAy \\ 1.532 * dAx - 1.254 * dAy \\ 1.532 * dAx - 1.254 * dAy \end{bmatrix}$$

-vector of quadrupoles for alpha function correction

Although the solution using the inverse matrix can be obtained in one step using (12), it cannot be used directly, since there are restrictions on the value of permissible currents in quadrupole lenses. Therefore, the algorithm is organized as an iterative cycle, in which at each iteration a check is made for the values of the quadrupole currents selected by the algorithm to be outside the limits of permissible values.

Next, the beta function correction algorithm looks like this, where the input vector X looks like this $[\beta_x, \beta_y, 0, 0]$:

corLimit - upper limit, new how much current can be set
corLimitLower - lower limit of how much current can be reduced

```
#X - the input vector of betas of beam parameters
#N - is the number of iterations
#cor result vector, which contains how much the initial value of the quadrupole gradients needs to be changed
#corp, cor_new - vectors for intermediate values
ROWS, COLS = ResponseMatrix.shape
cor = np.zeros(shape=(COLS))
corp = np.zeros(shape=(COLS))
cor_new = np.zeros(shape=(COLS))
for _ in range(N):
    for n in range(COLS):
        R = ResponseMatrix[:, n]
        Rinv = np.power(R.transpose() @ R, -1) * R.transpose()
        corp[n] = cor[n]
        cor_new[n] = Rinv @ X
        cor[n] = cor[n] + cor_new[n]
        if np.abs(cor[n]) > corLimit:
            cor[n] = corLimit * np.sign(cor[n])
            cor_new[n] = cor[n] - corp[n]
        if np.abs(cor[n]) < corLimitLower:
            cor[n] = corp[n]
            cor_new[n] = 0
        if cor[n] == corp[n]:
            cor_new[n] = 0
        X = X - R * cor_new[n]
corrPower2 = cor

return  corrPower2
```

These codes return a vector "corPower2" containing the currents to correct the beta function.

1. R = ResponseMatrix[:, n] - extracts the n column from ResponseMatrix
2. Rinv = np.power(R.transpose() @ R, -1) * R.transpose() - calculate the inverse matrix for n-th column
3. corp[n] = cor[n] - stores the current value of cor[n] for comparison later
4. cor_new[n] = Rinv @ X - we calculate the new quadrupole changes
5. cor[n] = cor[n] + cor_new[n] - update the cor with new quadrupole changes
6. If the n-th correction modulus is greater than corLimit, then it is limited by corLimit (with the sign preserved) and cor_new[n] is updated accordingly.
7. If the absolute value is less than corLimitLower, return the n-th value of cor[n] back to its last result in corp[n], and cor_new[n] set to zero.
8. If cor[n] is equal to corp[n], this means that there is no change in the quadrupole gradient, and cor_new[n] is set to zero.
9. Finally X = X - R * cor_new[n] - update the beta parameters with the new changes in the quadrupole gradients.
10. And we go through the main cycle until the values of the quadrupole gradient correction are within the acceptable values.

The given addition is needed to calculate the required currents for correcting the alpha function (corrPower1) and for changing the beta function (corrPower2):

$$C = corrPower1 + corrPower2 \tag{14}$$

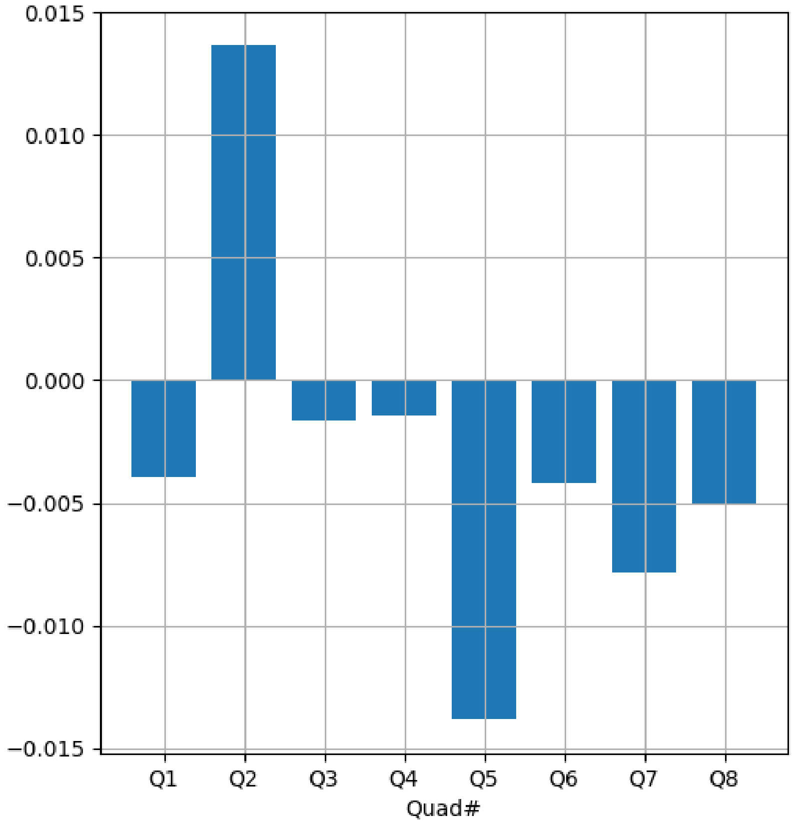

Figure 10 - Example of the algorithm's result

2.4 Channel matching management program

The programming language chosen to write the channel matching program was Python version 3.12.0. Python has a number of advantages that make it a convenient tool for this task:

1) the numpy library for working with matrices

2) the PySide library for creating a graphical interface and visualizing the results in the form of a graph.

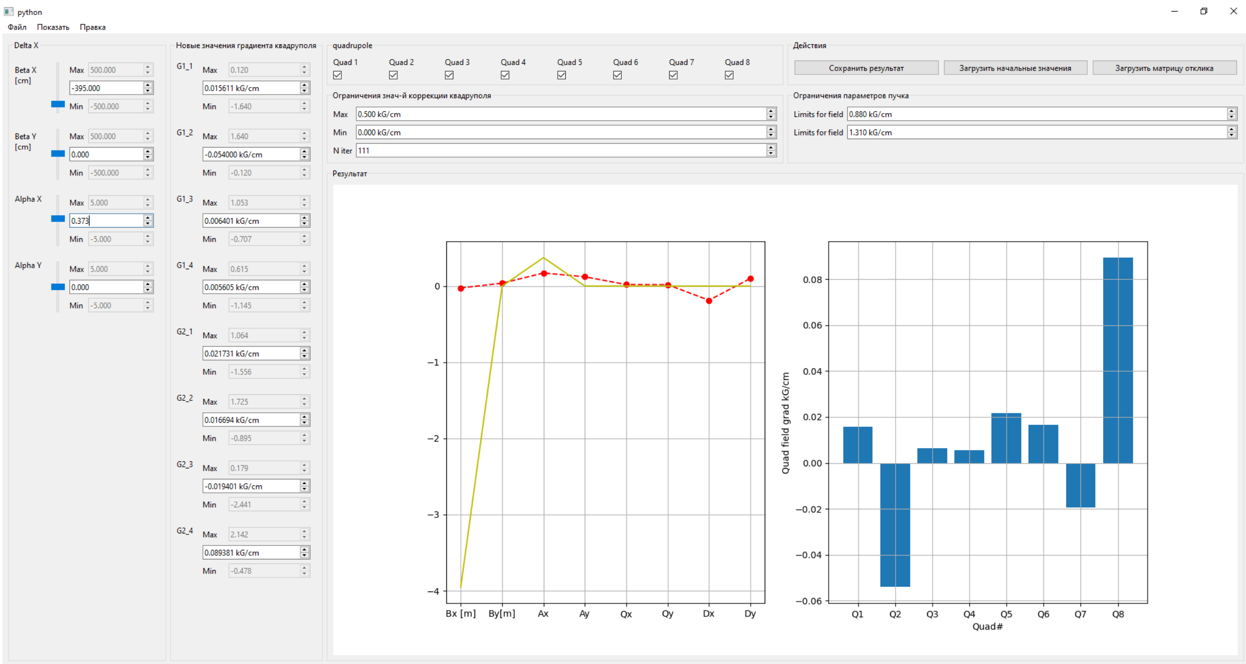

Figure 11 - Screenshot of the program interface

2.5 Example of using the program

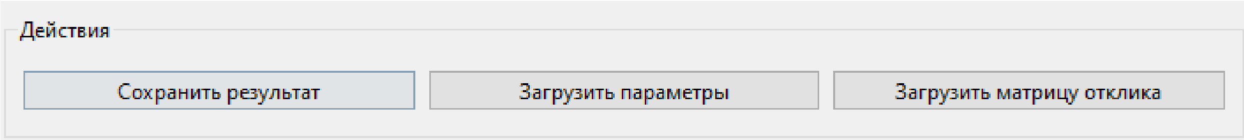

Figure 12 - Action buttons

The following buttons (Fig. 12) are responsible for:

- Save results — save results to desired path;
- Load parameters — load initial values of quadrupole field gradients and constraints on quadrupole field values;
- Load response matrix — load response matrix from a file.

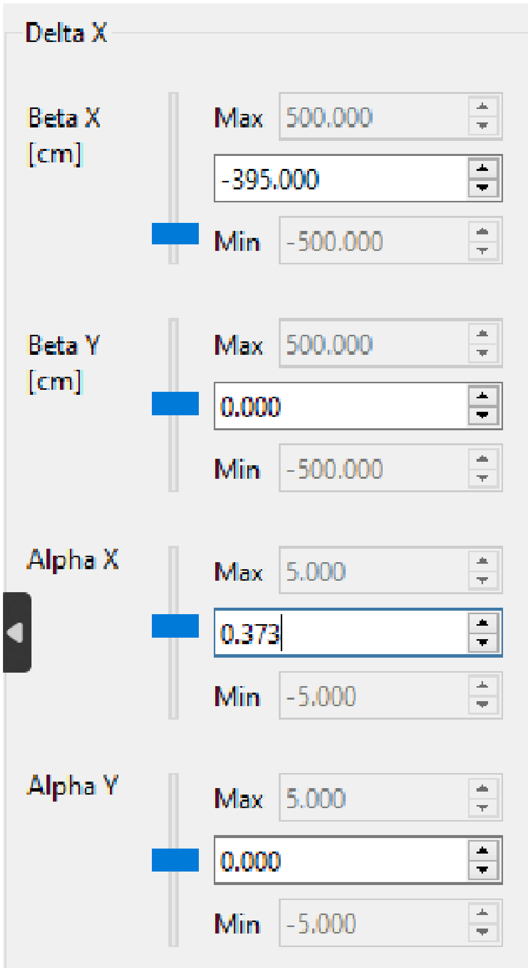

Figure 13 - Twiss Parameter input fields

In the window with the input fields of Twiss parameters (Fig. 13) the values are selected. When each value is changed, the algorithm works and the result is displayed in the gradient fields (Fig. 14) and in the graph (Fig. 15).

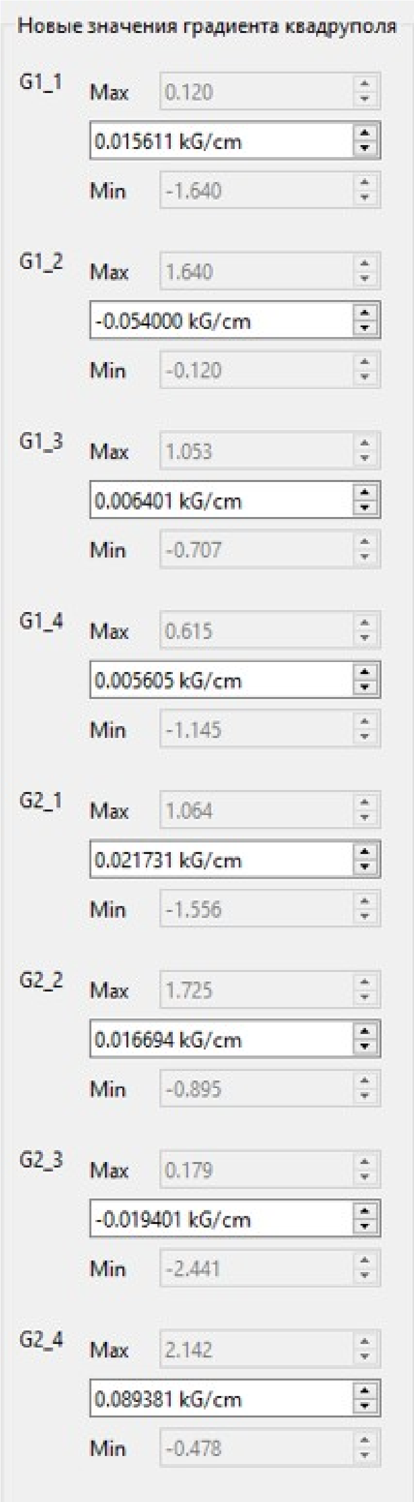

Figure 14 - Elements of the required vector of gradients of fields of quadrupole lenses

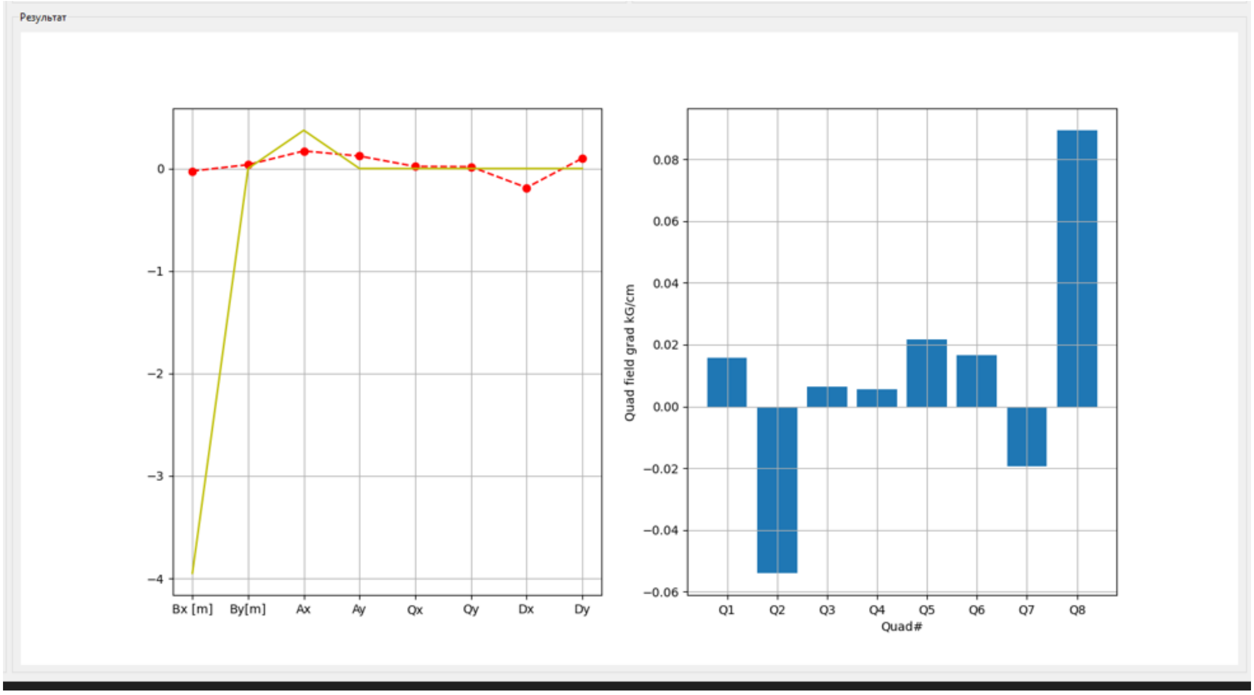

Figure 15 - Graph with the result of Twiss parameters correction (yellow - the entered parameters, red - the result of correction) and the values of quadrupole fields

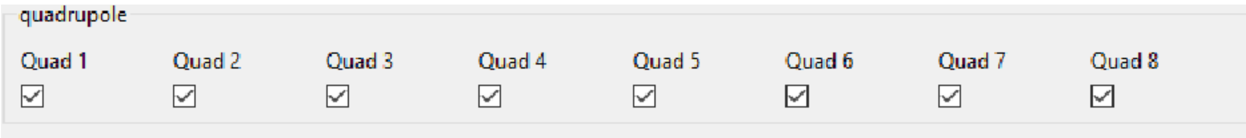

Figure 16 - Fields for turning on/off quadrupole magnets

If necessary, you can disable one or more channel quadrupole to adjust the Twiss parameters in the “Quadrupole” window (Fig. 16).

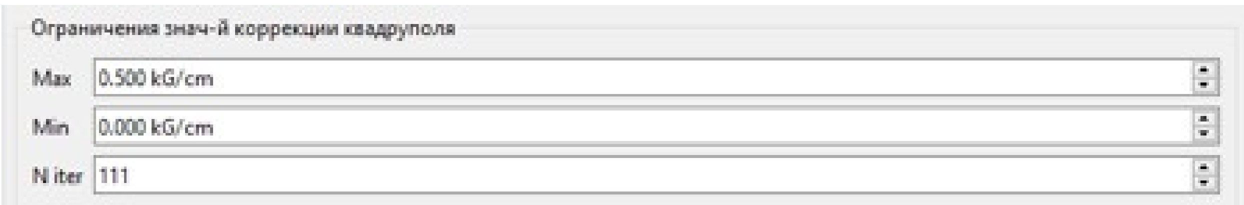

Figure 17 - Input fields for quadrupole correction limits

Conclusion

As a result of this work, an algorithm for correcting the beta function at the output of the Booster-Nuclotron channel and a program for matching the channel with the user interface were developed.

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