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FINAL REPORT ON THE

START PROGRAMME

Viscosity in an accelerated relativistic medium from the Unruh effect vs string theory bound

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Abstract

This project investigates the dissipative properties of an accelerated relativistic medium and their connection to the Unruh effect and effective black hole radiation. The thermodynamic properties in spaces with a horizon is one of the most discussed in modern fundamental physics. A notable 2005 string theory limit sets a minimum shear viscosity. We calculated viscosity in an accelerated frame for a photon medium, where no holographic description exists, treating the black hole horizon as a membrane of finite thickness. While the average viscosity meets the string theory limit, local values are described by a universal function that is independent of particle spin. Specifically, on the membrane surface, the ratio of local viscosity to local entropy is half the string theory limit. Importantly, this result is gauge-independent, with the positive contribution from gauge fixing exactly canceling the negative contribution from Faddeev-Popov ghosts.

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1 Introduction

How easy is it to imagine a four-dimensional Euclidean (not to mention five- or six-dimensional spaces) space, it is also difficult to imagine a body in Rindler space. Have you ever thought about the fact that the famous Hawking radiation has a lesser—known "brother" - the Unruh effect. In this study, we will calculate the ratio of viscosity to entropy using the KovtunSon-Starinets a method for a medium arising in the context of the Unruh effect during the interaction of a scalar massless field. We will analyze the value of the "local" viscosity and the identified phenomena.

Goal:

Of great interest is the question of the hydrodynamic properties of relativistic swirling liquids placed in a space with a horizon, such as the space of a black hole or space The Rindler. The aim of the work is to investigate issues related to entropy and viscosity in appropriate media, in particular, to verify the viscosity constraint from string theory. Formula, obtained in [Son09].

Tasks:

1. It is necessary to study the existing approaches to the description of dissipative effects in spaces with a horizon: − Calculation of viscosity and entropy for a scalar field in the Rindler space.

2. Calculation of viscosity and entropy for photons and comparison with the absolute limit on the ratio of shear viscosity to entropy predicted from string theory.

3. Comparison of "local" and global dissipative characteristics.

In 2005, the work of D.T. Son showed that there are no completely ideal liquids. Instead , there is a restriction from below on the ratio of viscosity to entropy. The corresponding result was obtained using string theory and the holographic approach, and is currently one of the most well-known and important predictions in string theory. In our work , we plan to investigate the validity of this prediction in a completely new system corresponding to quantum fields in an accelerated system the countdown. At the moment, only the literature has been considered scalar fields, at the same time string theory predicts the universality of this limit. Therefore, we want to investigate it for fields with different spins. Since accelerated quark-gluon plasma occurs in collisions of heavy ions at particle accelerators , the work is also of interest from the point of view of phenomenology.

Results:

1. The viscosity of photons in an accelerated reference frame at the Unruh temperature was calculated for the first time.

2. The ratio of the average viscosity to the average entropy for photons in a Minkowski vacuum in an accelerated medium has been clearly shown. This value is equal to $\frac{1}{4\pi}$, which corresponds to the limit derived from string theory. 3. It is shown that the ratio of local viscosity to local entropy on the membrane surface is two times less than the limit from string theory and is equal to $\frac{1}{8\pi}$.

4. The ratio of local viscosity to local entropy is found at an arbitrary distance from the membrane of the extended horizon for photons. It is clearly shown that this function is universal for massless particles with different spins.

5. Various approaches to calculating entropy are analyzed and it is shown that the thermodynamic definition through the pressure derivative is in agreement with the limit on the ratio of viscosity to entropy from string theory.

6. It is clearly shown that the viscosity of photons does not depend on the choice of gauge and there is a mutual compensation of the contributions of the members fixing the gauge and the Faddev-Popov ghosts. Thus, all the tasks were completed.

2 Unruh effect

A vacuum is not an empty space. It is filled with fields in which there are zero fluctuations, fluctuations. If the particle detector moves in a straight line at a constant speed, then it does not detect particles. However, the accelerated detector will begin to register particles, which is due to the fact that the vacuum in the accelerated frame of reference (in coordinates Rindler's vacuum), differs from the Minkowski vacuum. As a result, the detector will show the existence of a "thermal bath". The properties of this thermal medium are closely related to the existence of the horizon and are most well known It is the thermodynamic characteristics: energy, temperature, etc. The dissipative properties of this medium have been much less studied. Unruh radiation is similar to Hawking radiation, but it is not its analogue. In the second case, the particles will carry away the energy of the black hole. A qubit (a two-level system with two energy levels) with two energy states would be well suited as a detector. This way it will be able to show either the presence or absence of particles. The Unruh effect was theoretically predicted by a Canadian physicist By William Unruh in 1976 [Unr76].

3 Stretched and true horizon

For our calculations, it is worth introducing the cut off stretched horizon. It is 2+1-dimensional time-like surface located slightly outside the true horizon Because it has a non-singular induced metric, the stretched horizon provides a more tractable boundary on which to anchor external fields; outside a complicated boundary layer, the equations governing the stretched horizon are to excellent approximation the same as those for the true horizon. This view of a black hole as a dynamical time-like surface, or membrane, has been called the membrane paradigm [Par98]. Cutting off this horizon will allow us to get rid of divergences at infinities and remove the effects of Red and Blue displacement.

In our calculations, we will denote the thickness of the horizon l_c. Accordingly, integration by Rindler coordinates will be carried out from l_c to infinity.

4 Methods

After the last Superstring Revolution, it became necessary to explain the predicted effects using quantum field theory. Obtaining confirmation of predictions from the String theory using quantum field theory is an important and rare event; for example, testing the predictions of the String theory. The dissipative thermodynamic characteristics obtained from this theory can be verified using classical methods of field theory. Namely, we will consider the value of the viscosity-to-entropy ratio for media in the accelerated Rindler space, which is associated with the Unruh effect. If it is not difficult to find the entropy of the system, then it requires a lot of calculations. This work for a scalar massless field has already been done in [Chi10]. However, such a solution remains not obvious for other types of fields, due to the different values of entropy. We performed a calculation for a vector massless field using the Kubo formula, where we used the average value of the correlator of two energy-momentum tensors, and also used the point-splitting technique to eliminate uncertainties. In the end, we also turned to the theory of functions of complex variables, using Cauchy's theorem for residues. We also separately counted contributions from Maxwell's, gauge's, and ghost's tensor members to show their role in the calculations.

5 Calculations of viscosity

In our work, we will give in more detail the derivation of the viscosity value from the work [Chi10].We introduce the Rindler space, which is most often used in the study of the Unruh effect. The relation of Minkowski coordinates to Rindler coordinates is described by the following expressions: $z \to t\sinh(\beta) + z\cosh(\beta)$, $t = \rho\sinh(\eta)$, $z = \rho\cosh(\eta)$ So, the Rindler space metric has the form:

$$ds^2 = \rho^2 d\eta^2 - d\rho^2 - dx^2 - dy^2$$

Among the properties of the Rindler coordinates is the concept of the Rindler horizon, which can be considered as a real black hole. Let's assume that this horizon has a non-zero finite thickness l_c. Namely, it is a very thin membrane. [Par98] Now we can proceed to calculating the viscosity value, for this we use the Standard method of calculating transport coefficients – the Kubo formula [Kub57]. We need the Kubo Formula for shear viscosity [Son09]:

$$\eta = \lim_{\omega\to 0} \int_{l_c}^{\infty} \chi\, d\chi \int_{l_c}^{\infty} \chi'\, d\chi' \int_{-\infty}^{\infty} dx\, dy\, d\tau e^{i\omega\tau} < 0|\hat{T}_{xy}(\tau, x, y, \chi)\hat{T}_{xy}(0, 0, 0, \chi')|0 >_m \quad (1)$$

where $< 0|\hat{T}_{xy}\hat{T}_{xy}|0 >_m$ is the quantum average of the vacuum Minkowski from the product of two energy-momentum tensors. It contains information about the properties of the field – mass and spin. Specifically, in our work we will consider a vector massless field.

We introduce the formula of the propagator of a vector massless field in the pulsed form from [LL75], where ξ is gauge value.

$$\frac{-i}{(2\pi)^4} \int \frac{d^4 p e^{ip(x-y)}}{p^2} [\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)] \quad (2)$$

Let's write it in the coordinate representation, using positive-frequency Whiteman function [Shi80]

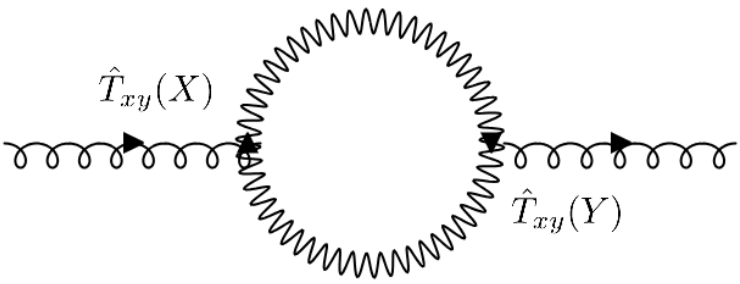
$$\frac{1}{8\pi^2 x^2} [\eta_{\mu\nu}(1+\xi) + \frac{2x^\mu x^\nu}{x^2}(1-\xi)] \quad (3)$$

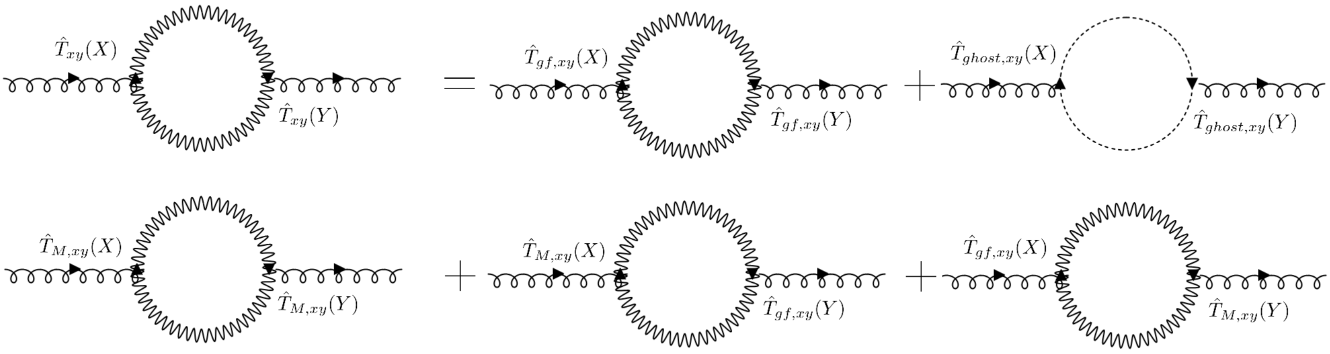
Consider the quantum average of the product of two energy-momentum tensors of our field (consider only the Maxwell terms, gauge terms and ghosts terms separately)

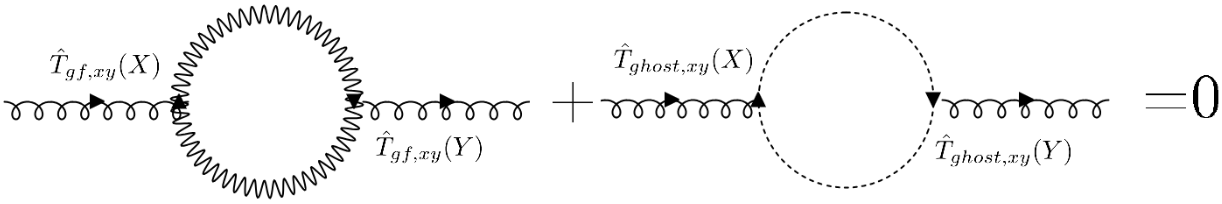
$$\begin{aligned}(-F^{\mu\alpha}F^{\nu}_{\alpha})(-F^{\lambda\beta}F^{\rho}_{\beta}) = &\left(\frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_2^{\nu}}A_{\alpha}A^{\alpha} - \frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial Y_{2\alpha}}A_{\alpha}A_{\nu} - \frac{\partial}{\partial Y_1^{\alpha}}\frac{\partial}{\partial X_2^{\nu}}A_{\mu}A^{\alpha} + \frac{\partial}{\partial Y_1^{\alpha}}\frac{\partial}{\partial Y_{2\alpha}}A_{\mu}A_{\nu}\right)\cdot \\ &\cdot\left(\frac{\partial}{\partial X_1^{\lambda}}\frac{\partial}{\partial X_2^{\rho}}A_{\beta}A^{\beta} - \frac{\partial}{\partial X_1^{\lambda}}\frac{\partial}{\partial Y_{2\beta}}A_{\beta}A_{\rho} - \frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial X_2^{\rho}}A_{\lambda}A^{\beta} + \frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_{2\beta}}A_{\lambda}A_{\rho}\right)\end{aligned} \tag{4}$$

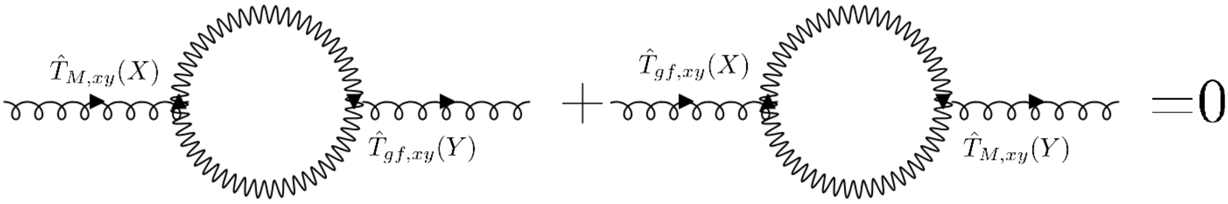
$$\begin{aligned}(-F^{\mu\alpha}F^{\nu}_{\alpha})(-F^{\lambda\beta}F^{\rho}_{\beta}) = &\frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_2^{\rho}}A_{\alpha}A^{\alpha}A_{\beta}A^{\beta} - \frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_{2\beta}}A_{\alpha}A^{\alpha}A_{\beta}A_{\rho} - \\ &-\frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_2^{\rho}}A_{\alpha}A^{\alpha}A_{\lambda}A^{\beta} + \frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_{2\beta}}A_{\alpha}A^{\alpha}A_{\lambda}A_{\rho} - \\ &-\frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_2^{\rho}}A_{\alpha}A_{\nu}A_{\beta}A^{\beta} + \frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_{2\beta}}A_{\alpha}A_{\nu}A_{\beta}A_{\rho} + \\ &+\frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_2^{\rho}}A_{\alpha}A_{\nu}A_{\lambda}A^{\beta} - \frac{\partial}{\partial X_1^{\mu}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_{2\beta}}A_{\alpha}A_{\nu}A_{\lambda}A_{\rho} - \\ &-\frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_2^{\rho}}A_{\mu}A^{\alpha}A_{\beta}A^{\beta} + \frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_{2\beta}}A_{\mu}A^{\alpha}A_{\beta}A_{\rho} + \\ &+\frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_2^{\rho}}A_{\mu}A^{\alpha}A_{\lambda}A^{\beta} - \frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_2^{\nu}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_{2\beta}}A_{\mu}A^{\alpha}A_{\lambda}A_{\rho} + \\ &+\frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_2^{\rho}}A_{\mu}A_{\nu}A_{\beta}A^{\beta} - \frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\lambda}}\frac{\partial}{\partial Y_{2\beta}}A_{\mu}A_{\nu}A_{\beta}A_{\rho} - \\ &-\frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_2^{\rho}}A_{\mu}A_{\nu}A_{\lambda}A^{\beta} + \frac{\partial}{\partial X_1^{\alpha}}\frac{\partial}{\partial X_{2\alpha}}\frac{\partial}{\partial Y_1^{\beta}}\frac{\partial}{\partial Y_{2\beta}}A_{\mu}A_{\nu}A_{\lambda}A_{\rho}\end{aligned} \tag{5}$$

After that, using Wick's theorem, we omit the disconnected terms, thereby moving on to the study of the one-loop Feynman diagram:









$$
\begin{aligned}
<(-F^{\mu\alpha}F^{\nu}_{\alpha})_1(-F^{\lambda\beta}F^{\rho}_{\beta})_2> = {} & \frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\lambda}<A_\alpha A_\beta>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\rho}<A^\alpha A^\beta> + \frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\lambda}<A^\alpha A_\beta>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\rho}<A_\alpha A^\beta> - \\
& -\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\lambda}<A_\alpha A_\beta>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b_\beta}<A^\alpha A_\rho> - \frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\lambda}<A^\alpha A_\beta>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b_\beta}<A_\alpha A_\rho> - \\
& -\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\beta}<A_\alpha A_\lambda>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\rho}<A^\alpha A^\beta> - \frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\beta}<A^\alpha A_\lambda>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\rho}<A_\alpha A^\beta> + \\
& +\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\beta}<A_\alpha A_\lambda>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b_\beta}<A^\alpha A_\rho> + \frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\beta}<A^\alpha A_\lambda>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b_\beta}<A_\alpha A_\rho> - \\
& -\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\lambda}<A_\alpha A_\beta>\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\rho}<A_\nu A^\beta> - \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\lambda}<A_\nu A_\beta>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\rho}<A_\alpha A^\beta> + \\
& +\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\lambda}<A_\alpha A_\beta>\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b_\beta}<A_\nu A_\rho> + \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\lambda}<A_\nu A_\beta>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b_\beta}<A_\alpha A_\rho> + \\
& +\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\beta}<A_\alpha A_\lambda>\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\rho}<A_\nu A^\beta> + \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\beta}<A_\nu A_\lambda>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\rho}<A_\alpha A^\beta> - \\
& -\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b^\beta}<A_\alpha A_\lambda>\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b_\beta}<A_\nu A_\rho> - \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\beta}<A_\nu A_\lambda>\frac{\partial}{\partial b^\mu}\frac{\partial}{\partial b_\beta}<A_\alpha A_\rho> - \\
& -\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\lambda}<A_\mu A_\beta>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\rho}<A^\alpha A^\beta> - \frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\lambda}<A^\alpha A_\beta>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\rho}<A_\mu A^\beta> + \\
& +\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\lambda}<A_\mu A_\beta>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b_\beta}<A^\alpha A_\rho> + \frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\lambda}<A^\alpha A_\beta>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b_\beta}<A_\mu A_\rho> + \\
& +\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\beta}<A_\mu A_\lambda>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\rho}<A^\alpha A^\beta> + \frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\beta}<A^\alpha A_\lambda>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\rho}<A_\mu A^\beta> - \\
& -\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b_\beta}<A_\mu A_\rho>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b^\beta}<A^\alpha A_\rho> - \frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\beta}<A_\mu A_\lambda>\frac{\partial}{\partial b^\nu}\frac{\partial}{\partial b_\beta}<A^\alpha A_\rho> + \\
& +\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\lambda}<A_\mu A_\beta>\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\rho}<A_\nu A^\beta> + \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\lambda}<A_\nu A_\beta>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\rho}<A_\mu A^\beta> - \\
& -\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\lambda}<A_\nu A_\beta>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b_\beta}<A_\mu A_\rho> - \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\lambda}<A_\nu A_\beta>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b_\beta}<A_\mu A_\rho> - \\
& -\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\beta}<A_\mu A_\lambda>\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\rho}<A_\nu A^\beta> - \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\beta}<A_\nu A_\lambda>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\rho}<A_\mu A^\beta> + \\
& +\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\beta}<A_\mu A_\lambda>\frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b_\beta}<A_\nu A_\rho> + \frac{\partial}{\partial b_\alpha}\frac{\partial}{\partial b^\beta}<A_\nu A_\lambda>\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b_\beta}<A_\mu A_\rho>
\end{aligned}
\tag{6}
$$

Here we also use the technique of point-sliping, to avoid ambiguities in expressions[Chi10]. Some members of the expression disappear when switching to the numerical values of the fields in question $\mu = \lambda = 1, \nu = \rho = 2$. Thus, all metrics will go to zero, except $\eta_{\mu\lambda}$ и $\eta_{\nu\rho}$. Which, in turn, are equal -1. Also, accordingly, changes b_μ, b_λ and b_ν, b_ρ на $-x$ и $-y$. We will also carry out substitutions related to $b^2 = t^2 - x^2 - y^2 - (z - z')^2 - i\epsilon\epsilon(t)$. Exactly, $\alpha = -t^2 + (z - z')^2 + i\epsilon\epsilon(t)$ и $-(x^2 + y^2) = -r^2$. So we move on to a spherical coordinate system and integrate it into the horizon plane:

$$< T^{12}T^{12} >= \frac{2}{\pi^4(-\alpha-r^2)^4} + \frac{4x^2}{\pi^4(-\alpha-r^2)^5} + \frac{4y^2}{\pi^4(-\alpha-r^2)^5} + \frac{16x^2y^2}{\pi^4(-\alpha-r^2)^6} \tag{7}$$

replacing x and y with $r\cos(\phi)$ and $r\sin(\phi)$

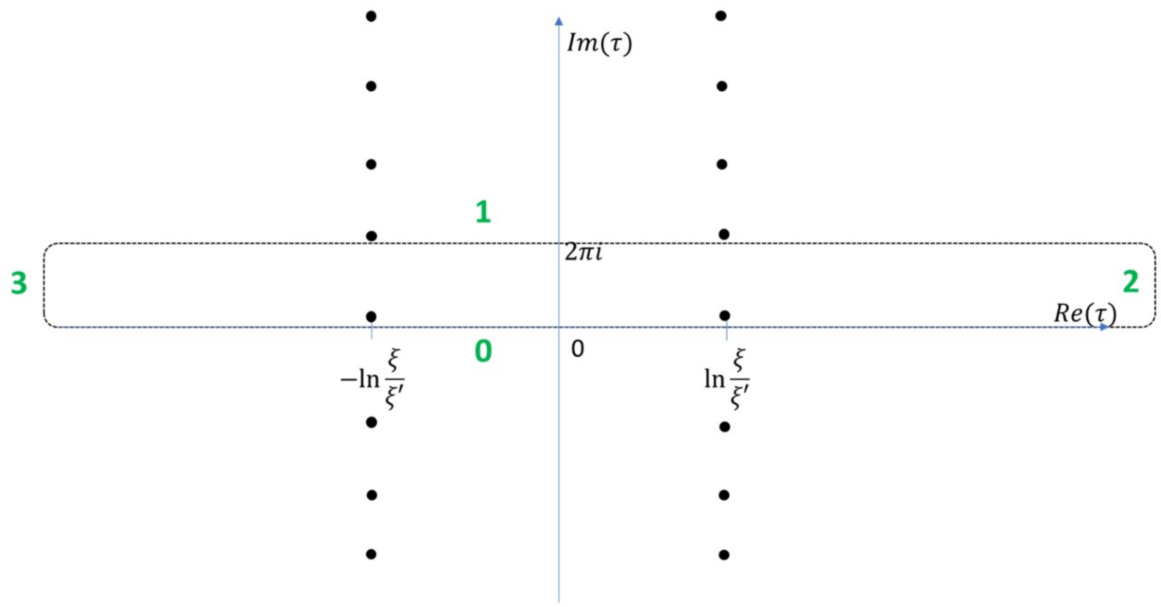
$$< T^{12}T^{12} >= \frac{2r\left(\alpha^2 - r^4\cos(4\phi)\right)}{\pi^4(\alpha+r^2)^6} \tag{8}$$

$$\int_{l_c}^{\infty}\int_{l_c}^{\infty} < T^{12}T^{12} > \, dx\, dy = \int_{l_c}^{\infty}\int_{0}^{2\pi} r < T^{12}T^{12} > \, d\phi\, dr = \frac{2}{5\alpha^3\pi^2} \tag{9}$$

Let's move on to the Rindler coordinates explicitly

$$\int_{-\infty}^{\infty} \frac{2e^{it\omega}}{5\pi^2(\chi^2+\chi'^2-2\chi\chi'\cosh(t))^3}\, d\tau \tag{10}$$

There are two poles of the expression, which, as a result of periodicity, turn into an infinite set of points concentrated along parallel lines. To calculate this integral, it is necessary to apply Cauchy's theorem, known from the Theory of functions of a complex variable. Creating a contour infinite along the axis of the abscissa, we will place two poles there. Let's set a rule for bypassing the contour and divide it into 4 parts, where 2 and 3 will immediately turn to zero. Let's integrate and take the limit:



$$\eta = \int_{l_c}^{\infty} \chi\, d\chi \int_{l_c}^{\infty} \chi'\, d\chi' \frac{-6\chi^4 + 6\chi'^4 + 4(\chi^4 + 4\chi^2\chi'^2 + \chi'^4)\ln\left(\frac{\chi}{\chi'}\right)}{5(\chi^2-\chi'^2)^5\pi^2} \tag{11}$$

$$\eta = -\int_{l_c}^{\infty} \chi'\, d\chi' \frac{-5l_c^4 + 4l_c^2\chi'^2 + \chi'^4 + 4(l_c^4 + 2l_c^2\chi'^2)\ln\left(\frac{l_c}{\chi'}\right)}{20(l_c^2-\chi'^2)^4\pi^2} \tag{12}$$

$$\eta = \frac{l_c^4 - \chi'^4 - 4l_c^2\chi'^2\ln\left(\frac{l_c}{\chi'}\right)}{40(\chi'^2-l_c^2)^3\pi^2} \tag{13}$$

The viscosity value for a vector massless field has the form:

$$\eta = \frac{1}{120\pi^2 l_c^2} \tag{14}$$

6 Gauge

We introduce the value of the energy-momentum tensor with gauge corrections and Faddeev-Popov ghosts, omitting the diagonal terms that turn to zero in our work. [LL75]

$$T^{\mu\nu}_{photon,gauge} = \frac{1}{\xi}(A^\mu \partial^\nu(\partial A) + A^\nu \partial^\mu(\partial A)) \tag{15}$$

Interestingly, the members of the species $\frac{\partial}{\partial b^\alpha}\frac{\partial}{\partial b^\beta}A^\alpha A^\beta$ for this propagator, they turn to zero, as a result, all fourth-order derivatives in our calculations are also zero. Despite this, the cross products of the Maxwell terms and the gauge ones, by themselves, do not turn 0; however, the sum of them is mutually compensated and gives zero.

$$< T^{12}_{photon,gauge} T^{12}_{photon,gauge} >= \frac{1}{2\pi^4(-\alpha - r^2)^4} + \frac{2x^2}{\pi^4(-\alpha - r^2)^5} + \frac{2y^2}{\pi^4(-\alpha - r^2)^5} + \frac{16x^2y^2}{\pi^4(-\alpha - r^2)^6} \tag{16}$$

replacing x and y with $r\cos(\phi)$ and $r\sin(\phi)$

$$< T^{12}_{photon,gauge} T^{12}_{photon,gauge} >= \frac{r\left((\alpha - r^2)^2 - 4r^4\cos(4\phi)\right)}{2\pi^4(\alpha + r^2)^6} \tag{17}$$

$$\int_{l_c}^{\infty}\int_{l_c}^{\infty} < T^{12}_{gauge} T^{12}_{gauge} > \, dx\, dy = \int_{l_c}^{\infty}\int_{0}^{2\pi} r < T^{12}_{gauge} T^{12}_{gauge} > \, d\phi\, dr = \frac{1}{15\alpha^3\pi^2} \tag{18}$$

$$\int_{-\infty}^{\infty} \frac{e^{it\omega}}{15\pi^2(\chi^2 + \chi'^2 - 2\chi\chi'\cosh(t))^3}\, d\tau \tag{19}$$

$$\eta = \int_{l_c}^{\infty} \chi\, d\chi \int_{l_c}^{\infty} \chi'\, d\chi' \frac{-3\chi^4 + 3\chi'^4 + 2(\chi^4 + 4\chi^2\chi'^2 + \chi'^4)\ln\left(\frac{\chi}{\chi'}\right)}{15(\chi^2 - \chi'^2)^5\pi^2} \tag{20}$$

$$\eta = -\int_{l_c}^{\infty} \chi'\, d\chi' \frac{-5l_c^4 + 4l_c^2\chi'^2 + \chi'^4 + 4(l_c^4 + 2l_c^2\chi'^2)\ln\left(\frac{l_c}{\chi'}\right)}{120(l_c^2 - \chi'^2)^4\pi^2} \tag{21}$$

$$\eta = \frac{l_c^4 - \chi'^4 - 4l_c^2\chi'^2\ln\left(\frac{l_c}{\chi'}\right)}{240(\chi'^2 - l_c^2)^3\pi^2} \tag{22}$$

The value of the contribution to viscosity for a vector massless field from the gauge terms of the energy-momentum tensor:

$$\eta = \frac{1}{720\pi^2 l_c^2} \tag{23}$$

It is interesting to note that the expression does not depend on ξ.

7 Ghosts

It is also necessary to take into account the Faddeev-Popov ghosts. In the case of multiplying these terms with Maxwell and gauge terms, we immediately get zero due to their commutativity properties. However, there remain 4 terms obtained by multiplying the wind terms. [DIK09]

$$T^{12}_{photon,ghosts} = -\frac{1}{\xi}(\partial^\mu \bar{c} \partial^\nu c + \partial^\nu \bar{c} \partial^\mu c) \tag{24}$$

where c is The Lagrangian of ghosts: [DIK09].

$$c = \frac{-i}{(2\pi)^4}\int \frac{d^4p e^{ip(x-y)}}{p^2}[\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}(1-\xi)] \tag{25}$$

$$\begin{aligned} &< \frac{1}{\xi^2}(\partial^\mu \bar{c} \partial^\nu c + \partial^\nu \bar{c} \partial^\mu c)(\partial^\lambda \bar{c} \partial^\rho c + \partial^\rho \bar{c} \partial^\lambda c) >= \\ &= -\frac{1}{4\pi^4(-\alpha - r^2)^4} - \frac{x^2}{\pi^4(-\alpha - r^2)^5} - \frac{y^2}{\pi^4(-\alpha - r^2)^5} - \frac{8x^2y^2}{\pi^4(-\alpha - r^2)^6} \end{aligned} \tag{26}$$

The expression turns out to be equal to a similar calculation for a scalar massless field multiplied by 2. replacing x and y with $r\cos(\phi)$ and $r\sin(\phi)$

$$< T^{12}_{photon,ghosts} T^{12}_{photon,ghosts} >= -\frac{r\left((\alpha - r^2)^2 - 4r^4\cos(4\phi)\right)}{2\pi^4(\alpha + r^2)^6} \tag{27}$$

$$\int_{l_c}^{\infty}\int_{l_c}^{\infty} < T^{12}_{ghosts} T^{12}_{ghosts} > dx\, dy = \int_{l_c}^{\infty}\int_{0}^{2\pi} r < T^{12}_{ghosts} T^{12}_{ghosts} > d\phi\, dr = -\frac{1}{15\alpha^3\pi^2} \tag{28}$$

$$-\int_{-\infty}^{\infty} \frac{e^{it\omega}}{15\pi^2(\chi^2 + \chi'^2 - 2\chi\chi'\cosh(t))^3}\, d\tau \tag{29}$$

$$\eta = -\int_{l_c}^{\infty} \chi\, d\chi \int_{l_c}^{\infty} \chi'\, d\chi' \frac{-3\chi^4 + 3\chi'^4 + 2(\chi^4 + 4\chi^2\chi'^2 + \chi'^4)\ln\left(\frac{\chi}{\chi'}\right)}{15(\chi^2 - \chi'^2)^5\pi^2} \tag{30}$$

$$\eta = \int_{l_c}^{\infty} \chi'\, d\chi' \frac{-5l_c^4 + 4l_c^2\chi'^2 + \chi'^4 + 4(l_c^4 + 2l_c^2\chi'^2)\ln\left(\frac{l_c}{\chi'}\right)}{120(l_c^2 - \chi'^2)^4\pi^2} \tag{31}$$

$$\eta = -\frac{l_c^4 - \chi'^4 - 4l_c^2\chi'^2\ln\left(\frac{l_c}{\chi'}\right)}{240(\chi'^2 - l_c^2)^3\pi^2} \tag{32}$$

The value of the contribution to viscosity for a vector massless field from the members of the Fadeev-Popov ghosts of the energy-momentum tensor:

$$\eta = -\frac{1}{720\pi^2 l_c^2} \tag{33}$$

It is worth noting that it does not depend on ξ as well as the gauge terms. Moreover, the expression is equal to it in modulus, as a result of which the contribution from the gauge parts and the parts of the ghosts turns to zero.

8 Calculations of entropy and the final limit

It is known from statistical physics [LL80]:

$$s_{loc} = -\frac{1}{V}\frac{\partial F}{\partial T} = \frac{\partial p}{\partial T} \tag{34}$$

Where T is temperature. In our case, this is the Unruh temperature equal to $\frac{a}{2\pi}$, where a is the acceleration of the system, which can also be considered in Rindler coordinates.

The pressure for spin 1:

$$p = \frac{1}{3}\left(\frac{\pi^2 T^4}{15} + \frac{T^2}{6\chi^2} - \frac{11}{240\pi^2\chi^4}\right) \tag{35}$$

[Dow94] Given $Z = \frac{1}{|a|}$, $a = \frac{1}{\chi}$, we get the entropy values:

$$s_{loc} = \frac{1}{15\pi\chi^3} \tag{36}$$

$$s = \frac{1}{30\pi l_c^2} \tag{37}$$

We obtain the limit of the ratio of shear viscosity to entropy

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{38}$$

It is also noteworthy that this limit does not depend on the value l_c.

Note that the correspondence with the limit of string theory is obtained using [BDS24], whereas using the expressions for entropy from [Dow94] we would get a completely different expression.

9 Local viscosity and limit

We obtain the values of the so-called "local" viscosity. The value turns out to be non-obvious and more cumbersome than the value of the integral viscosity:

$$\frac{\eta_{loc}}{s_{loc}} = \frac{3\chi(\chi^4 + 4\chi^2 {l_c}^2 - 5{l_c}^4 - 4{l_c}^2(2\chi^2 + l_c^2)\ln\left(\frac{\chi}{l_c}\right))}{4\pi(\chi^2 - l_c^2)^4} \tag{39}$$

As you can see, this expression is derived from the Maxwell terms of the tensor. The values obtained from the ghosts and gauge members are equal and opposite in sign.

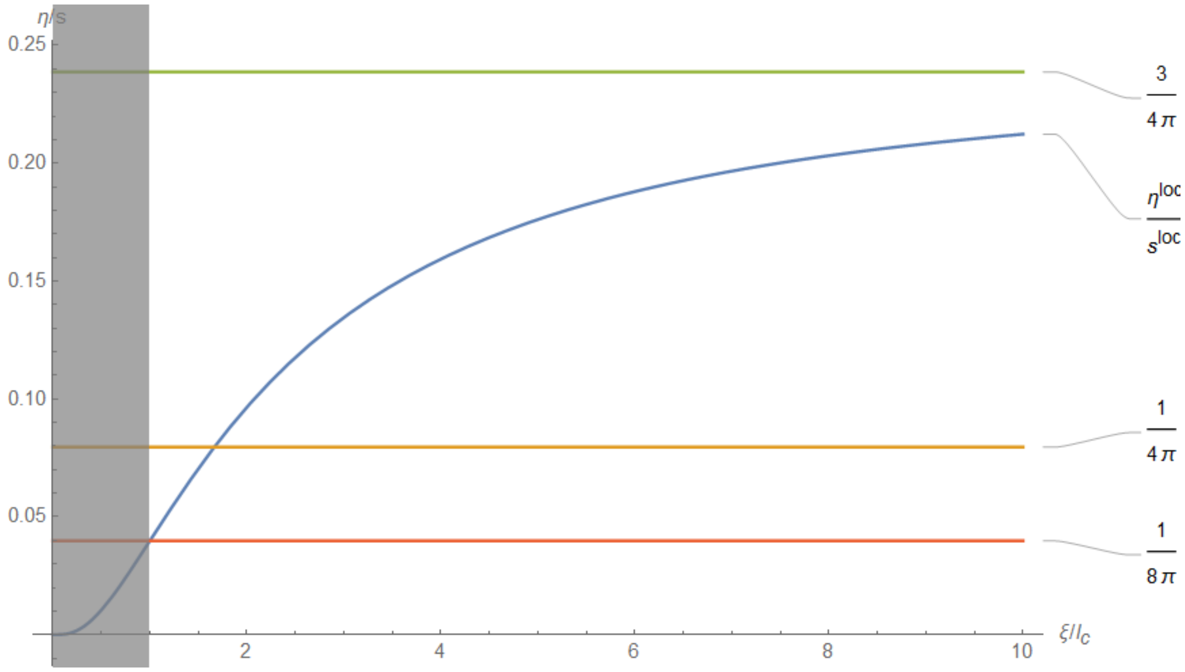
At a value below $1{,}66 l_c$, the value of the "local" viscosity falls below the minimum value for global viscosity; and at $l_c \to 0$ (which corresponds to the membrane surface)It is equal to $\frac{1}{8\pi}$, that is, half of the minimum value.

At $\chi \to \infty$ (away from the horizon), the viscosity value tends to $\frac{3}{4\pi}$.

When the thickness of the l_c membrane changes at constant χ. With a value of $l_c \to 0$, which corresponds to the assumption of the absence of a membrane, we get $\frac{3}{4\pi}$.Also, this value will not depend on χ!This indicates a constant viscosity value at any distance of many large membrane thicknesses from the membrane.

The appearance of a distance dependence is a direct indication of the presence of a membrane of finite thickness, and the scale at which the ratio of viscosity to entropy varies markedly determines the thickness of the membrane.

despite the fact that the viscosity is equal to $\frac{3}{4\pi}$, this is compensated by its drop on scales of the order of the membrane thickness below and $\frac{1}{4\pi}$; which on average results in $\frac{3}{4\pi}$.



On the graph, the gray area indicates the membrane area, the distance from the stretched to the true horizon.

10 Conclusions

Let's summarize and list the results obtained.

1. The viscosity of photons in an accelerated reference frame at the Unruh temperature was calculated for the first time.

2. The ratio of the average viscosity to the average entropy for photons in a Minkowski vacuum in an accelerated medium has been clearly shown. This value is equal to $\frac{1}{4\pi}$, which corresponds to the limit derived from string theory. 3. It is shown that the ratio of local viscosity to local entropy on the membrane surface is two times less than the limit from string theory and is equal to $\frac{1}{8\pi}$.

4. The ratio of local viscosity to local entropy is found at an arbitrary distance from the membrane of the extended horizon for photons. It is clearly shown that this function is universal for massless particles with different spins.

5. Various approaches to calculating entropy are analyzed and it is shown that the thermodynamic definition through the pressure derivative is in agreement with the limit on the ratio of viscosity to entropy from string theory.

6. It is clearly shown that the viscosity of photons does not depend on the choice of gauge and there is a mutual compensation of the contributions of the members fixing the gauge and the Faddev-Popov ghosts. Thus, all the tasks were completed.

11 Research prospects

The prediction for the viscosity-to-entropy ratio from string theory is universal. At the same time, it is not obvious from the calculation we have carried out - the calculation directly depends on which fields we are considering. Therefore, it is of interest to check the universality of the limit by considering higher spins, as well as massive fields. It can be noted that this work uses characteristics directly related to the properties of particles. In this paper, we have considered massless particles with spin 1, which fully corresponds to the behavior of such a particle as a photon. If we take into account the mass of the particle, then we get the values for the carriers of the weak interaction of $W^{\pm}$ и Z^0 bosons. Then we can see the difficulties, in particular, the effects of quantum chromodynamics will give us values for gluons and gluon plasma. Subsequently, the addition of quarks will give us a value for the quark-gluon plasma, which is most observed in quantum effects, and which will be easier to detect in the case of the Unruh effect experiment. To summarize, we can say that we are ready to publish an article on this topic based on the above calculation results.

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