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FINAL REPORT ON THE START PROGRAMME

Beam parameters restoration at the NICA accelerator complex – based on the beam position monitor data

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Abstract

This research focuses on optimizing the diagnostic system of the NICA accelerator complex - a powerful facility for studying nuclear matter and elementary particle physics. A specific emphasis is placed on the superconducting booster, which plays a crucial role in generating intense beams of heavy ions. The work involves developing and refining mathematical models to describe the motion of charged particles within the booster, as well as analyzing data signals from the beam position monitor to reconstruct/restoration beam parameters. The optimization of data analysis algorithms in the diagnostic system aims to enhance its efficiency. The solutions obtained through this research can be also applied to other accelerator complexes.

Contents

Introduction

Accelerator complexes like NICA (Nuclotron-based Ion Collider fAcility) represent complex and powerful tools for studying the structure of nuclear matter and conducting various scientific research in the field of elementary particle physics. Achieving design parameters and ensuring the stable operation of the accelerator complex are critically important as they determine the luminosity in physical experiments and the possibility of conducting unique scientific investigations.

The booster is a crucial component in the NICA accelerator complex which plays a critical role in obtaining the required intensity of heavy ion beams. The main tasks of the booster include:

- Accelerating ion beams with minimal losses due to residual gas;
- Shaping the required phase space of the beam using an electron cooling system;
- Fast extraction of the accelerated beam and injection into the Nuclotron.

Obtaining the design parameters of accelerators is impossible without precise tuning and optimization of their systems. This requires the development and improvement of mathematical models describing the dynamics of charged particle motion, as well as the development of signal analysis methods obtained from diagnostic equipment.

The aim of this work is to investigate and optimize the store and analysis of data from the diagnostics systems of the NICA injection complex.

In this work, the main focus is on the booster of the NICA accelerator complex. The key tasks of the work are as follows:

1. Developing mathematical models describing the dynamics of charged particle motion in booster lattice.
2. Analyzing data from the beam position monitor in the booster to restore beam parameters such as size, position, shape, and momentum spread.
3. Optimizing the efficiency of data analysis algorithms for the diagnostic system.
4. Testing the developed algorithms on a set of experimental data.

Solving the set tasks will enable the application of the developed methodologies to other facility within the NICA accelerator complex.

1. Introduction to the System

1.1. The NICA accelerator complex

The NICA accelerator complex (Nuclotron-based Ion Collider fAcility) (See Fig. 1) is a complex system comprising an injector, a Booster, an upgraded Nuclotron accelerator, and a Collider consisting of two storage rings [7]. This is one of the flagship projects of the JINR (Joint Institute for Nuclear Research) and one of the Mega-science projects which located in the territory of the Russian Federation.

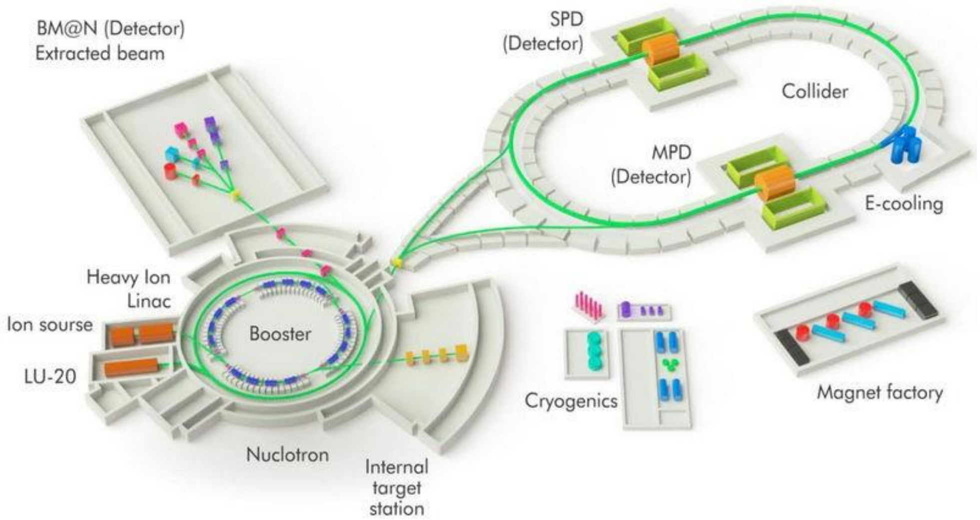

Figure 1 - Schematic view of the NICA accelerator complex [4].

The most important fundamental research directions in this field include:

1. Nature and Properties of Strong Interactions: Studying the strong interactions between the elementary constituents of the Standard Model of particle physics, namely quarks and gluons.
2. Phase Transition Search: Searching for signs of a phase transition between hadronic matter and QGP (Quark-Gluon Plasma), as well as exploring new states of baryonic matter.
3. Study of Strong Interaction and QGP Symmetry: Investigating the fundamental properties of strong interactions and QGP symmetry [4].

The heavy ion injection chain consists of a source, a linear accelerator (HILAC), and a superconducting Booster. The Booster is located within the Synchrophasotron yoke and is designed to accelerate heavy ions to an energy of approximately 600 MeV/nucleon (maximum magnetic rigidity of 25 Tm).

The upgraded Nuclotron will provide proton, deuteron, and heavy ion beams with a maximum magnetic rigidity of up to 38.5 Tm. The maximum designed luminosity of the Collider when working with heavy ions is $10^{27}\ cm^{-2}s^{-1}$.

In the initial stages, experiments are planned with symmetric collisions of heavy ions such as ^{197}Au, ^{208}Pb and ^{209}Bi. The collision energy $\sqrt{s}$ ranges from 7 to 9.46 GeV/nucleon, with an energy of $\sqrt{s}$ 9.2 GeV/nucleon allowing for comparisons with results from the STAR experiment at the RHIC collider in BNL. In the long term, NICA aims to achieve the maximum collision energy of Au + Au collisions with √s up to 11 GeV/nucleon.

Moreover, NICA will enable the production of polarized proton and deuteron beams with a center-of-mass energy of up to 27 GeV and a luminosity of $10^{32}\ cm^{-2}s^{-1}$. The Collider's magnetic system is based on superconducting dual-aperture magnets, with a maximum magnetic field induction in dipole magnets of 18 Tm. Electron and stochastic cooling systems are also planned to maintain luminosity in the Collider.

After discussing and analyzing the NICA project, let's move on to a more detailed description of the key components of the diagnostic system. The primary focus is on the Beam Position Monitor (BPM), its role, and functionality. Next, we will delve into the structure and working principle of the BPM.

1.2. Device and principle of operation of the beam position motors

The BPM is a device designed to observe and measure the instantaneous position and shape of a charged particles beam in the accelerator, as shown in Figure 2. These data are crucially important for the precise tuning of the accelerator and ensuring the stability of beam circulation, which directly impacts the quality and outcomes of experiments.

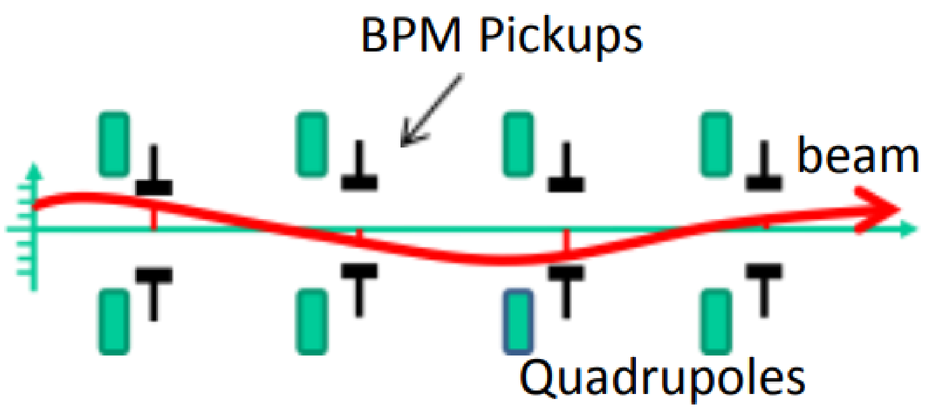

Figure 2 - Schematic representation of the Beam Position Monitor [4].

The operation of the BPM is based on various principles of beam position measurement: electrical, magnetic, or electromagnetic. The choice of a specific sensor types depends on the accelerators characteristics and the requirements placed on it.

The working principle of the BPM relies on the setup symmetry. The symmetry of the beam's center of mass position is determined by a pair of symmetrically placed electrodes [4], as shown in Figure 3.

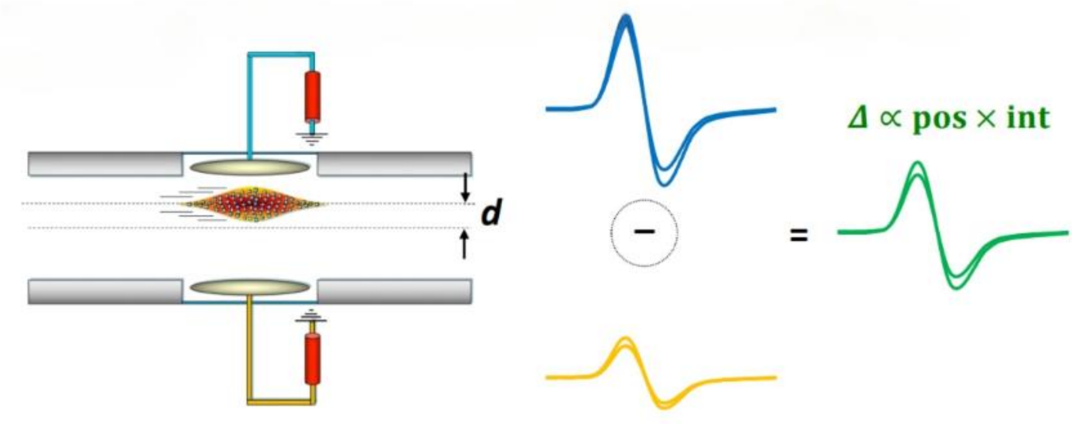

Figure 3 - Working principle of the beam position monitor [4].

Interpreting signals from sensors is a crucial step in the process of beams diagnostics. The acquired signals can be presented in analog or digital form and contain information about the distribution of particles within the beam in terms of coordinates, energies, and other characteristics. This ultimately enables the development of methods to enhance the quality of the beam.

A beam position monitoring system typically consists of an array of sensors placed along the trajectory of the particle beam. These sensors detect the passage of charged particles, providing information about their positions at specific sections of the accelerator. Analyzing this data enables the determination of the beams spatial location, as well as its shape and dimensions.

In conclusion, the BPM stands as a pivotal instrument in ensuring the stable and high-precision operation of charged particle accelerators. Accurate information about the beam's position and its characteristics allows for the control and optimization of its movement, which is a critical requirement for successfully tuning the setup's design parameters and carrying out experimental and scientific research endeavors.

1.2.1. Signal Analysis using the Beam Position Monitor

The analysis of the signal from the beam position monitor depends on the type of sensor used and its operating principle. Primarily, the interpretation of the signal is aimed at determining the current position of the charged particle beam within the accelerator.

Various methods of signal interpretation can be applied for different types of sensors used in beam position monitors. Below are a few examples of signal interpretation for some sensor types:

1. Ionization detectors employ vacuum chambers filled with gas through which the beam passes. The interaction of the beam's charged particles with the gas leads to gas ionization, resulting in the formation of ions and electrons. Detectors register these ions and electrons, and signal interpretation involves analyzing the time of flight and the distribution of these charged particles to determine the beam's position [1].

2. Electromagnetic detectors are based on measuring changes in the magnetic field caused by the passage of the beam's charged particles. Detectors register the magnetic field changes and interpret them to determine the beam's position in space [1].

3. Semiconductor sensors: The response of a semiconductor sensor depends on the charged particles passing through the sensor material. Signal interpretation involves analyzing the amplitude and time of flight of the charged particles, enabling the determination of their position [1].

The underlying principle of signal interpretation involves analyzing the acquired data and its correlation with coordinates, time of flight, or other characteristics of the charged particles. Data processing algorithms and signal interpretation methods can be diverse and depend on the specific sensor type and its operational principle.

2. Charged Particle Motion

The dynamics of charged particle motion in accelerators describe their interactions with facility components and among themselves. Analyzing this dynamics allows for finding optimal settings to achieve design parameters. Conducting such analysis is impossible without data on the current beam parameters, which can be measured or reconstructed using various methods, as well as mathematical models describing the dynamic processes of charged particle motion.

Several key models describing motion dynamics are introduced, including phase space, transverse and longitudinal motion frequencies, betatron functions, resonances, and chromaticity, etc.

2.1. Phase volume

In the context of particle accelerators, the phase space represents a multi-dimensional space where each measurable physical quantity related to particles (such as coordinates and momenta) corresponds to one of the coordinates in this space. For a system of N particles in three-dimensional space, the phase space is described by a 2N-dimensional phase space [3].

The formula for calculating the phase space in the case of transverse (vertical and horizontal) beam motion is as follows:

$$V = \sigma_x * \sigma_y * \sigma_{p_x} * \sigma_{p_y}, \quad (1)$$

where σ_x is the standard deviation of the horizontal position, σ_y is the standard deviation of the vertical position, σ_{p_x} is the standard deviation of the horizontal momentum, and σ_{p_y} is the standard deviation of the vertical momentum.

Representing particle behavior through phase space allows describing their positions and motions within an accelerator. It plays a crucial role in the analysis and design of accelerators as it helps understand how particle motion in the accelerator will be stable and how particles will interact with focusing elements and with each other.

Changes in the shape or size of the phase space can indicate various anomalies or nonlinearities in the accelerator's operation, which can impact the beam's structure and characteristics.

2.2. Betatron tunes

When a particle moves within the lattice structure of an accelerator, a deviation in its transverse coordinate or momentum from the design value results in the emergence of betatron oscillations around the equilibrium orbit. The linear betatron oscillations of a particle in a cyclic accelerator are described by the Hill's equation in the accompanying coordinate system [1]:

$$x'' + K_x(z)x = 0, \qquad y'' + K_y(z)y = 0, \tag{2}$$

where $x' = \frac{dx}{dz}, y' = \frac{dy}{dz}, K_x(z) = \frac{1}{B\rho}\frac{\partial B_y}{\partial x} + \frac{1}{\rho^2}, K_y(z) = \frac{1}{B\rho}\frac{\partial B_y}{\partial x} + \frac{1}{\rho^2},$ B – magnetic field, ρ – radius of curvature of the trajectory. Here, x corresponds to the horizontal plane, and y corresponds to the vertical plane.

The solution, for instance, in the vertical plane, can be found in the form of two independent solutions [3]:

$$y_1(z) = e^{\frac{i\varphi z}{L}} p_1(z), \qquad y_2(z) = e^{-\frac{i\varphi z}{L}} p_2(z),$$

where φ is the accumulated horizontal φ_x or vertical φ_y betatron phase, which is determined by the following relation:

$$\cos\varphi = \frac{1}{2} traceM(s)$$

$p_1(z)$ and $p_2(z)$ are periodic functions of z, and $M(s)$ is the transition matrix [3]:

$$p_i(z + L) = p_i(z), \qquad i = 1{,}2,$$

$$M(s) = M(s + \frac{L}{s})$$

where L is the length of the accelerator period.

The temporal variation of transverse coordinates and momentum is periodic but not sinusoidal. Therefore, the spectrum of their oscillations will contain a set of the betatron frequencies harmonics.

The horizontal Q_x or vertical Q_y betatron frequency in a cyclic accelerator is defined as the number of betatron oscillation periods occurring within one turn:

$$Q = \frac{\varphi(z + C) - \varphi(z)}{2\pi} = \frac{1}{2\pi}\int_z^{z+C} \frac{dz}{\beta(z)}, \tag{3}$$

where β is the corresponding beta function, C is the accelerator's perimeter [1].

Betatron frequencies are crucial parameters of an accelerator, greatly influencing its effective operation. The stability of beam motion, luminosity in colliders, and the brightness of synchrotron radiation sources largely depend on the working point's position (its betatron frequencies) in the betatron resonance plane. Consequently, measuring betatron frequencies is among the primary diagnostic procedures during facility startup.

The integer part of the betatron frequency can be easily determined based on the number of periods of wave-like orbit distortion caused by local magnetic field perturbation.

The fractional part of the betatron frequency can be measured through spectral analysis of an array of coherent betatron oscillations, detected by a beam position monitor on each turn.

2.3. Synchrotron Motion

Synchrotron motion is a type of particle motion in electromagnetic fields along the longitudinal direction. It's important to note that in synchrotron motion, the charged particle is accelerated.

For efficient particle acceleration and achieving high energies, maintaining resonance between the particle's revolution frequency and the changing magnetic field is crucial. This process is known as phase synchronization. The essence is that the magnetic field within the accelerator changes over time at a specific frequency. When the particle's revolution frequency aligns with this field-changing frequency, the particle gains extra energy with each revolution. Consequently, the particle accelerates during each loop until it reaches the desired energy.

However, synchrotron motion is accompanied by radiation, which leads to the particle losing energy. This synchrotron radiation is a result of the charged particle's acceleration and change in its direction within the magnetic field. Radiation can limit the maximum energy achievable in the accelerator and may require additional efforts to reduce these losses, such as using special magnetic systems or accelerator design.

Synchrotron motion of particles without acceleration also holds significance in modern particle physics and accelerator technology. It allows for creating bunched particle beams for research and applications across various scientific and technological fields.

2.4. Resonances

Chromaticity is associated with the dependency of the transverse betatron tunes of particle oscillations coincide with characteristic frequencies of perturbing influences. Resonance can lead to undesirable effects, such as beam instability, which can significantly impact accelerator performance and the quality of conducted scientific research.

To gain a more detailed understanding of resonance phenomena in accelerators, let's examine the synchronous frequency ω_s in the booster, which determines the moment when charged particles synchronously move with the variable electric field of the accelerator's accelerating station. This frequency is related to the particle's charge q, mass m, as well as the magnetic induction B_0 in the booster's magnetic fields.

When the frequency of longitudinal particle oscillations aligns with the characteristic revolution frequency of particles around the accelerator's ring, longitudinal resonances occur. This can lead to the instability of beam motion, an increase in beam dimensions, and particle loss.

2.5. Chromaticity

Chromaticity is associated with the dependency of the transverse betatron frequencies of a beam on its energy. This phenomenon can influence the stability of motion for such particles and lead to a reduction in the intensity of the accelerated beam. Therefore, controlling and correcting chromaticity is also important to ensure the stability of accelerator operation.

The dependence of beam focusing by the magnetic structure of the accelerator on the longitudinal momentum p is characterized by chromaticity ξ. Non-zero chromaticity leads to a shift in the betatron frequency of an off-momentum particle ($p = p_0 + \Delta p$). In a first-order approximation, the dependence ΔQ(Δp) can be considered linear:

$$\Delta Q = \xi \frac{\Delta p}{p}, \tag{5}$$

By introducing a small increment in the frequency of the RF accelerating system's master oscillator and thereby changing the revolution frequency of the beam f_0 by Δf, the relative change in longitudinal momentum becomes:

$$\frac{\Delta p}{p} = -\frac{1}{\eta}\frac{\Delta f}{f_0}, \tag{6}$$

where $\eta = \alpha - {}^{1}/_{\gamma^2}$; α is the orbit expansion coefficient; γ is the relativistic factor.

Thus, linear chromaticity can be determined by measuring the shift in the betatron frequency ΔQ as a function of the given increment in the beam's revolution frequency Δf:

$$\xi = -\frac{\eta \Delta Q f_0}{\Delta f}. \tag{7}$$

In the case of linear magnetic optics, the natural chromaticity is determined by the focusing force of quadrupole lenses:

$$\xi = -\frac{1}{4\pi}\int_{z}^{z+C} K(z)\beta(z)dz, \tag{8}$$

where $K(z)$ is the corresponding focusing parameter; $\beta(z)$ is the beta function.

Natural chromaticity is usually compensated by sextupole magnets, where a single sextupole of length l changes the chromaticity by an amount:

$$\xi = \pm\frac{1}{4\pi}D\beta K_2 l, \qquad (9)$$

where D is the dispersion function at the location of the sextupole; $K_2 = \frac{1}{B\rho}\frac{\partial^2 B_y}{\partial x^2}$. The plus sign corresponds to the plane of beam rotation [1].

2.6. Non-synchronous Particle

Nonsynchronous particles are particles that deviate from the equilibrium phase of the accelerating voltage, and they will be denoted by the index "s." Their motion does not precisely follow the phase trajectory of synchronous particles, and these deviations can be induced by external influences, inadequate accelerator tuning precision, or other factors.

Any other particle will then be defined by its deviation from equilibrium [3]:

Revolution frequency:	$f_r = f_{rs} + \Delta f_r$ $(\omega = \omega_s + \Delta\omega)$
RF phase:	$\phi = \phi_s + \Delta\phi$
Momentum:	$p = p_s + \Delta p$
Energy:	$E = E_s + \Delta E$
Azimuthal angle:	$\theta = \theta_s + \Delta\theta$

The azimuthal angle is connected to the longitudinal position through $ds = rd\theta$. In one complete revolution, this angle changes by 2π, whereas the radiofrequency phase changes by an amount of $2\pi h$, where h is the harmonic number. Therefore:

$$\Delta\phi = -h\Delta\phi$$

Furthermore, since $\theta = \int \omega dt$:

$$\Delta\omega = \frac{d}{dt}(\Delta\theta) = -\frac{1}{h}\frac{d\phi}{dt}$$

and from the definition of the slip factor $\eta = \frac{p}{f_r}\frac{df_r}{dp}$:

$$\Delta p = -\frac{p_s}{h\eta\omega_s}\phi$$

Additionally, during each revolution, a particle gains energy:

$$(\Delta E)_{turn} = e\,\hat{V}sin\phi,$$

where $\hat{V}$ is a slowly varying function of t [3].

It's worth mentioning that for any invariant of motion, there exists a relationship between the maximum energy and the maximum phase deviations. However, deriving it analytically is often difficult without specific assumptions. For instance, in the case of small-amplitude oscillations, the invariant is defined simply:

$$\frac{\phi'^2}{2} + \Omega_s^2 \frac{\Delta\phi^2}{2} = cte$$

which leads to, in the case of ultrarelativistic electrons [3]:

$$\Delta phi_{max} = \frac{\alpha h}{Q_s}\left(\frac{\Delta E}{E_s}\right)_{max}$$

Analyzing the behavior of nonsynchronous particles and their parameters provides a deeper understanding of their motion within accelerators. These interrelationships are crucial in developing correction methods to ensure the stability and efficiency of accelerators in the presence of nonsynchronous particles.

3. The practic

3.1. Aim

The aim of this study is to investigate and optimize the operation of the diagnostics system in the NICA injection complex, with a primary focus on the accelerator booster. The main goal of the study is to develop and refine mathematical models, analyze beam position monitor signals, and optimize data analysis algorithms to ensure the efficient functioning of the accelerator system and the achievement of design parameters.

The successful completion of the stated objectives will not only ensure the effective and stable operation of the booster within the NICA accelerator complex but also establish methodologies and approaches that can be applied and adapted to other facilities and accelerators.

3.2. Choice of Programming Language

The choice of a programming language for signal analysis tasks is based not only on convenience and work efficiency but also on the availability of a rich set of tools for scientific computations and data processing. For this task, the programming language Python, version 3.7 and higher, was selected.

Python possesses several advantages that make it a convenient tool for signal analysis:

1. The SciPy library provides a wide range of functions for scientific and engineering calculations, including signal processing, optimization, interpolation, and various other data analysis tasks.

2. Data analysis libraries like NumPy and pandas offer efficient storage, manipulation, and analysis of large volumes of data.

3. Visualization libraries such as Matplotlib, Seaborn, and Plotly enable the creation of scientific plots with rapid rendering.

4. Modularity and extensibility allow easy integration of third-party libraries and modules specialized for signal analysis.

5. An active community provides access to a broad range of knowledge, resources, and tools for solving diverse tasks.

Choosing Python version 3.7 and higher is justified by the fact that modern libraries and tools most comprehensively support the latest language versions, ensuring optimal performance and functionality for signal analysis.

3.3. Data

In this work, the following data are used, which are read from the Booster BPM .

The data is stored in a binary file, and each record consists of four numerical values representing data from four different BMP plates, as shown in Figure 4: right, top, left, bottom. Each set of four values corresponds to one time step, within a specific measurement interval of \ $(250 * 10^{6})^{-1}$ s. The data type in the file is $int16$, implying that each value from the pickup represents an integer using 16 bits for representation. This could correspond to an analog or digital measurement of signals on the Booster.

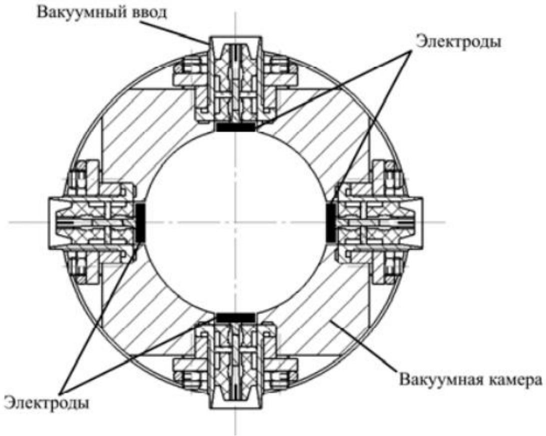

Figure 4 - Transverse Section of the Beam Position Monitor.

It is worth noting that each set of four values corresponds to one time sample, within a specific measurement interval of $(250 * 10^{6})^{-1}\ s$.

In addition, the data type in the int16 file implies that each pickup value is represented by an integer, using 16 bits for representation, which can correspond to analog or digital signal measurements on the booster.

Let's visualize the data, see Figure 5.

Figure 5 – Pickup Plate Data.

3.4. Data processing

The Fourier Transform is one of the fundamental methods for signal analysis and finds broad applications in various fields, including audio and video processing, radio engineering, medical diagnostics, and more. Let's explore the principles of the direct and inverse Fourier transforms, as well as their key properties.

The direct Fourier transform is a mathematical operation that translates a periodic signal from the time domain to the frequency domain.

The discrete Fourier transform (DFT) is the most practically used form [2]:

$$X_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n \cdot e^{-i2\pi\frac{kn}{N}} \qquad (10)$$

Where X_k is the complex value in the frequency domain for the k-th frequency, x_n is the value of the signal in the time domain for the n-th time sample, and N is the number of time samples (signal length).

The inverse Fourier transform allows the reconstruction of the original signal from its frequency representation. This is a crucial tool for signal processing and synthesis.

The mathematical formula for the inverse discrete Fourier transform (IDFT) is:

$$x_n = \sum_{k=0}^{N-1} X_k \cdot e^{i2\pi\frac{kn}{N}} \qquad (11)$$

Where x_n is the value of the signal in the time domain for the n-th time sample (reconstructed value), and X_k is the complex value in the frequency domain for the k-th frequency.

In this research work, direct and inverse Fourier transformations played a pivotal role in the analysis and processing of signals obtained from the Booster. For instance, these transformations were employed to accurately determine signal periods and identify the distribution of pulse deviations.

It's essential to emphasize that various normalization factors were applied during the implementation of these algorithms. These factors played a significant role in enhancing the quality of signal processing, ensuring more accurate and reliable analysis results.

3.5. Center of Mass Calculation

The main purpose of the Beam Position Monitor (BPM) is to determine the center of mass coordinates of the beam. To ascertain these coordinates, cumulative and differential resonances are utilized for each of the sensor axes (BPMs). The obtained results for each revolution are presented in the form of histograms in Figure 6. Analysis of the results reveals a dispersion of readings with a standard deviation of 1.53 mm for x and 0.86 mm for y. This outcome encompasses not only result repeatability but also the influence of beam injection errors. To mitigate the latter, it is necessary to conduct an analysis after the damping of coherent betatron oscillations induced by injection errors.

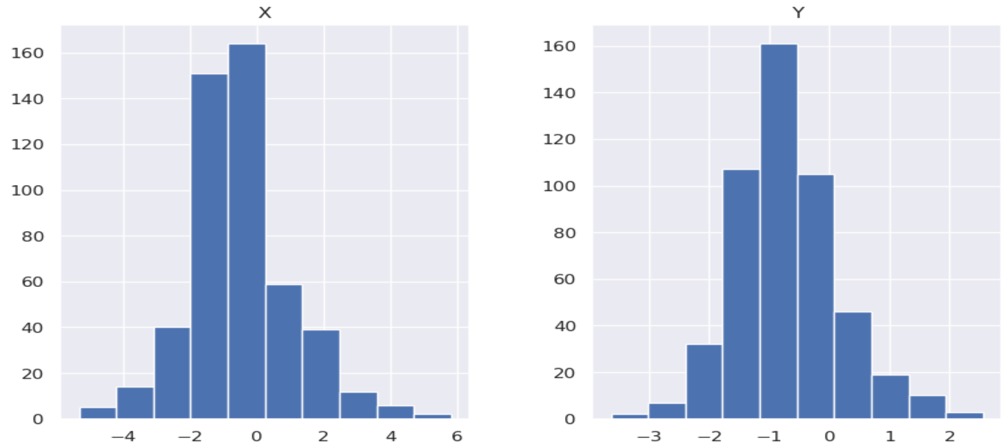

Figure 6 – Turn-by-turn Distribution of Center of Mass Coordinates

Analyzing Figure 6, it can be concluded that the mean values along each axis are slightly shifted in the negative direction. This result indicates that the particle beam is not moving precisely at the center of the BPM. Likely, there is some displacement that could be attributed to various factors such as perturbation of the closed beam orbit, injection errors, or external "noise" influences on the system.

Let's analyze how the center of mass position behaves during the initial revolutions. The analysis reveals damping of coherent oscillations in the center of mass of the beam during the initial ≈ 100 revolutions, as shown in Figure 7, caused by injection errors and the influence of the natural chromaticity of the magnetooptical system.

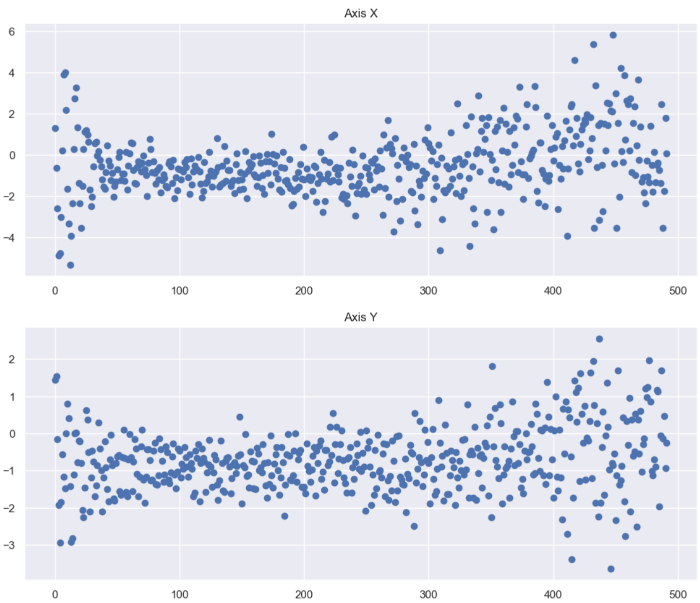

Figure 7 - Center of Mass Coordinates for Each Axis.

To determine the oscillation parameters, we will perform an approximation of the obtained results using a decaying oscillation function of a specified form:

$$y = Ae^{-\beta t}\cos(\omega x + \phi) + b \tag{12}$$

In this context, the parameter b takes on an additional significance. Its introduction is motivated by the need to compensate for the closed orbit offset from the BPM center and possible imperfections in BMP calibration, in order to achieve a more accurate approximation. The result of the approximation is shown in Figure 8.

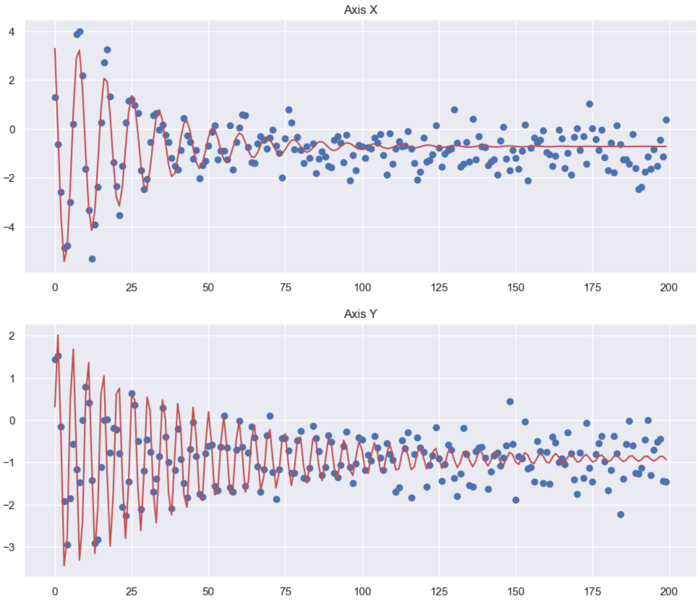

Figure 8 – Approximation of Center of Mass Coordinates for Each Axis.

An effective approximation process was achieved through the use of the SciPy library with the "optim" module. This tool allowed us to determine the optimal values for the parameters A, β, ω, ϕ, and b, ensuring the best fit of the approximating function to the data.

3.6. Recovery of Original Parameters

One of the important aspects in this research work was the restoration of the original signal shapes. The main factors leading to signal shape distortion include the limited bandwidth of the amplifier, delays in the signal line, and so on.

3.6.1. Determination of Chromaticity

Visual analysis of the plots in Figure 8 allows us to observe that the decaying oscillations reach their completion approximately: 75 revolutions in one plane and 150 in the other. The parameters of the decaying oscillations obtained in section 3.4 enable the determination of an important parameter – the natural chromaticity. To calculate it, the following steps are taken:

1. Perform the direct Fourier transform for the center of mass position coordinates over 2^N revolutions. This allows for the Discrete Fourier Transform (DFT) to be carried out and their spectrum determined.

2. Then, the spectrum is centered around the zero (revolving) frequency by subtracting it from all frequency components of the spectrum. This helps to focus on positive and negative frequencies relative to zero.

3. The next step involves filtering out the low frequencies responsible for the offset of oscillations from zero. This is necessary to retain only the high frequencies of interest for analysis.

4. Frequencies are then selected from zero to half the Nyquist frequency range [6], i.e., from 0 to 0.5. This corresponds to the left side of the spectrum where the frequencies of interest are presumed to be located. Alternatively, amplitude transformation is performed, and frequencies are selected from half the frequency range to the Nyquist frequency, 1.0. This represents the right side of the spectrum, which might also contain the fractional parts of the frequencies of interest.

The calculation results are presented in Figure 9.

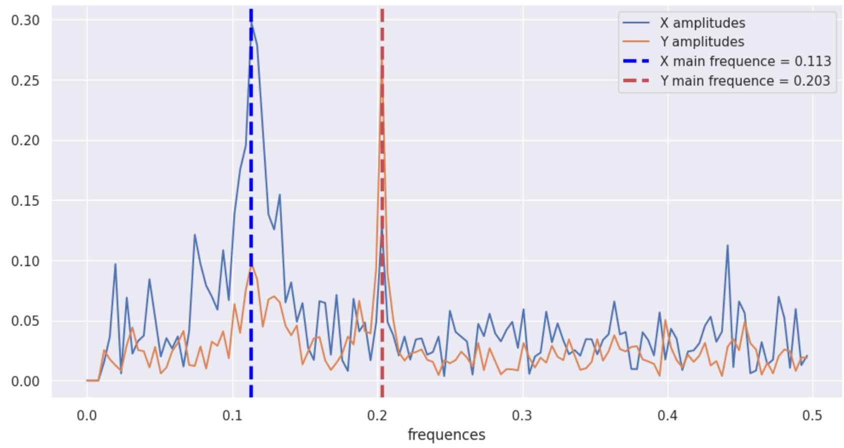

Figure 9 – Signal Spectrum Image with Highlighted Main Frequencies.

To conduct a comparative analysis between the objective function and the theoretical model, specific parameters have been defined. Parameters describing chromaticity were automatically obtained in the previous stage, acting as amplitudes in the formula (12) mentioned above. The comparison is presented in Figure 10. It should be noted that the initial function is negative, so the smoothing is performed using a function that cannot be negative.

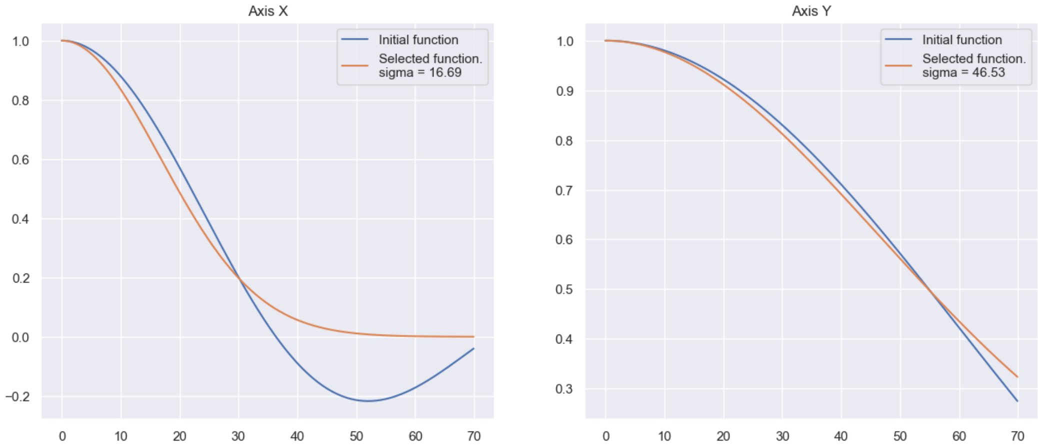

Figure 10 - Comparison of Results.

This stage of analysis allows us to assess the correspondence between the considered characteristics and the theoretical model that describes the process. The chromaticity parameters determined in the scope of this study can hold significant value for understanding the system's behavior. The obtained values are provided in Table 1:

Table 1. Chromaticity Parameters

	Axis X	Axis Y
Chromaticity	5.5	2.2

3.6.2. Frequency Response Restoration

The pickup has limitations that restrict the passage of low frequencies, which can significantly distort the shape of the measured signal. To address this issue, the restoration of the transfer function of the signal is considered to obtain the amplitude-frequency response (AFR) and the actual signal waveform.

For this purpose, based on the construction of the signal transmission and measurement system, the required form and parameters of the transfer function are chosen. The results of calculations of amplitudes and phases of the restored transfer function of the differentiating element for the frequency range of interest are presented in Figure 11.

The transfer function is a crucial tool in signal processing and the study of dynamic systems. It describes the system's influence on the input signal across various frequency ranges, accounting for and modifying both the amplitude and phase of the signal. In this context, the transfer function is utilized for correcting the AFR to adjust amplitude levels of signals in different frequency ranges. This correction helps eliminate distortions and ensures more accurate signal restoration and analysis. The form of the obtained transfer function in this work is as follows:

$$H(f) = \left(\frac{2j\,\pi\,f\,\tau_{FT}}{1 + 2j\pi f \tau_{FT}}\right)^{p} \tag{13}$$

where: $H(f)$ – transfer function, f – frequency, τ_{FT} – time constant of the transfer function, equal to $2.2 * 10^{-6}$, p – order of the function, equal to 1.

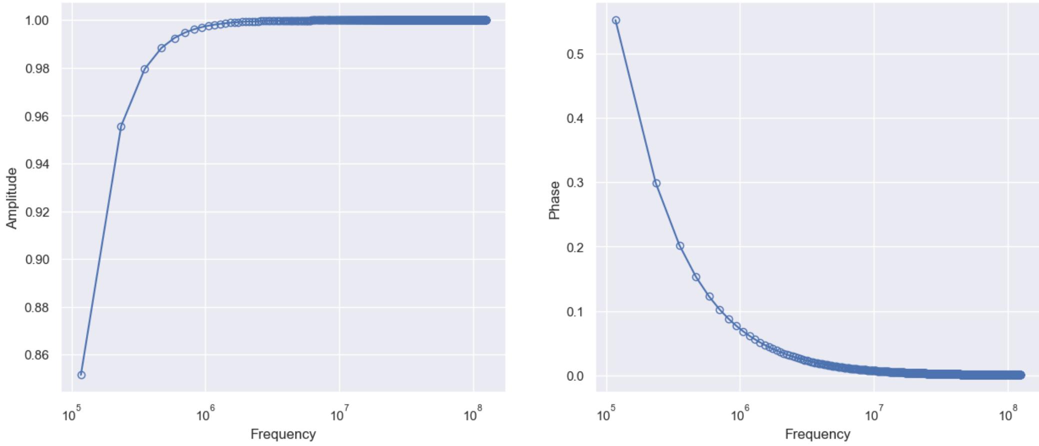

Figure 11 – Restored amplitude-frequency response (AFR) and phase-frequency response (PFR) based on the derived transfer function

3.6.3. Signal Summation Restoration

In the context of the work, another crucial step was undertaken – the restoration of the cumulative signal through its segmentation into periods, followed by subsequent correction. The purpose of this method is to enhance the accuracy and detail of the signal, while minimizing the influence of potential distortions and noise. The results of this procedure, including the restoration of the cumulative signal through periods and subsequent correction, were visualized in Figure 12.

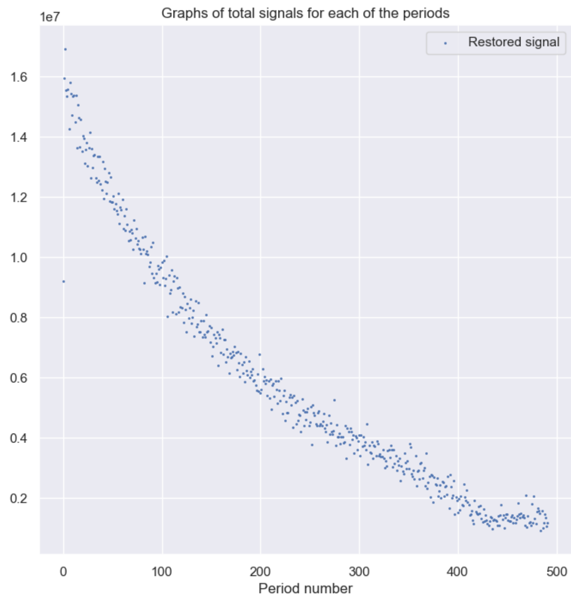

Figure 12 – Restored Total Signal over Periods.

The procedure of restoring the overall signal through its segmentation into periods involves segmenting the signal into smaller sections. This means that each period can be analyzed and restored more effectively independently of the others.

Following this operation, a series of corrections are applied sequentially with the aim of aligning and improving the amplitudes, as well as correcting discretization. Correction involves various methods, such as averaging smoothing, noise filtering, and so on.

3.6.4. Calculation of Momentum Deviation Distribution

In this study, one of the key aspects was the computation of the distribution of pulse deviations for particles, which is a significant parameter in analyzing the dynamics of particle motion within the setup. The aim of this procedure was to establish characteristics of the distribution. The results of this computation enable a more detailed assessment of the contribution of each component and its influence on the overall signal behavior. It should be noted that the expected outcome is to achieve a normal distribution form after applying the algorithm described below.

An iterative algorithm was employed to compute the distribution by determining the amplitudes of impulse components within a signal. Initially, the signal is discretized based on the impulses, after which the signal's shape is identified to create a distribution matrix. Utilizing this matrix of impulse deviation distributions along with corresponding initial amplitudes, the algorithm goes through multiple iterations, adjusting the amplitudes for each component to best match the signal. The algorithm iteratively fine-tunes the amplitudes of the impulse constituents to minimize the disparity between the computed signal and the input signal. The process involves computing intermediate values, subtracting initial distributions, and updating amplitudes based on normalized contributions. The ultimate result is a vector of amplitudes representing the distribution of impulse deviations. This approach precisely determines the contribution of each component to the overall signal and yields the distribution of impulse deviations. The outcome is presented in Figure 13.

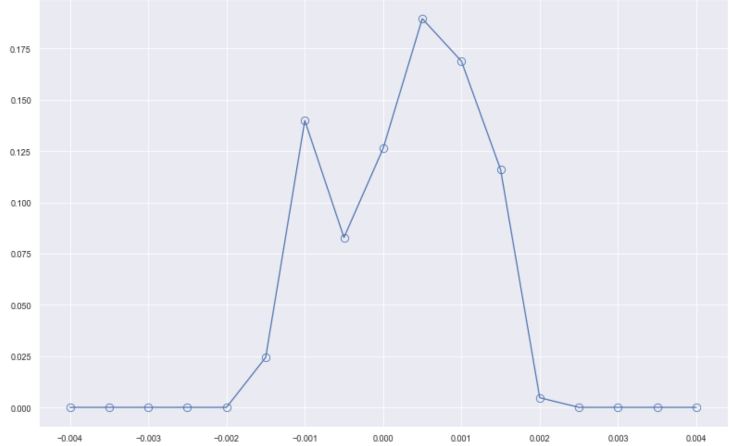

Figure 13 – Momentum Deviations Distribution.

An important step in the analysis of this signal was the application of a methodology involving the approximation of the signal's distribution shape with a parabola, followed by correction. The approximation procedure, based on this methodology, encompasses a set of methods aimed at computing and conducting a detailed analysis of signal parameters, relying on its amplitude characteristics and frequency components.

As a result of this analysis, it was possible to determine the adjusted values of the fundamental frequency and the parameters of the parabolic approximation. The obtained results are presented in Figure 14, where the impact of approximation and correction on the shape and structure of the signal is vividly illustrated.

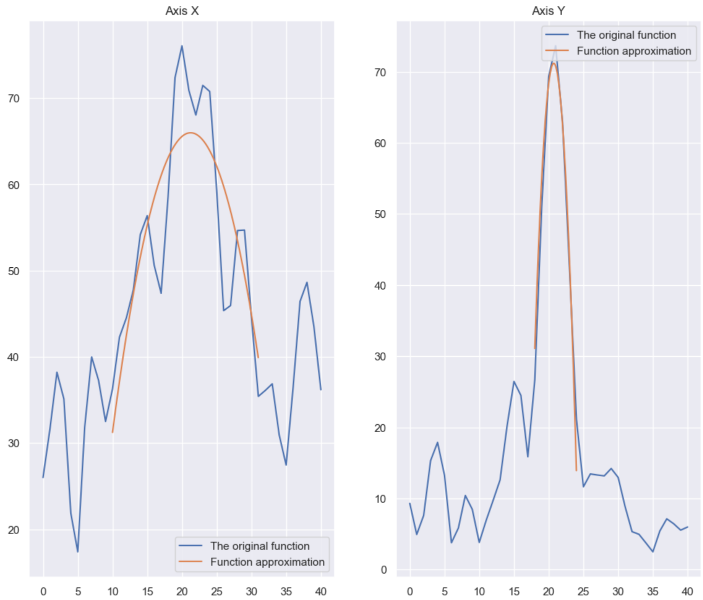

Figure 14 – Signal Approximation and Correction

This allowed for a more precise determination of distribution characteristics and the identification of hidden features, which is of crucial importance for a more accurate data analysis and interpretation of results.

Conclusion

This study implemented the reconstruction of parameters for a beam of charged particles in the NICA accelerator complex using signals from the beam position monitor. To achieve this, mathematical models of particle dynamics in accelerators were developed and refined, enabling the prediction of beam behavior and optimization of parameters to meet set objectives.

An analysis of signals obtained from the beam position monitor was conducted to control the shape, size, and momentum spread of particles in the beam. This provides crucial information for scientists and researchers, enabling the maintenance of required beam characteristics and ensuring high precision and stability in accelerator operation.

Furthermore, highly efficient signal processing algorithms were employed in the study, enabling more accurate and reliable data about beam parameters and its motion.

As a result of this work, successful restoration of charged particle beam parameters in the NICA accelerator complex was achieved. Software was developed for real-time signal processing, ensuring stability and high accuracy in its operation.

In the future, for further enhancement of this work, attention should be directed towards the following aspects: development and refinement of mathematical models for beam tomography, advancement of machine learning methods, etc.

References

1. Diagnostics of Charged Particle Beams in Accelerators / Ed. by corresponding member of the Russian Academy of Sciences N. S. Dikanskogo. Novosibirsk: Parallel, 2009. 294 p.

2. Discrete Fourier Transform. Educational Manual / V.P. Candidov et al. – Moscow: Faculty of Physics, Moscow State University, 2019. – 88 p, : ill. ISBN 978-5-8279-0179-2.

3. CERN. (1994). Introduction to Accelerator Physics (CERN-94-01-v1).

4. Monitoring and control in accelerators [Electronic resource]: inf. resource // [site]. - Ed. Jun 01 2019 - URL: https://indico.cern.ch/event/773082/contributions/3567657/attachments/1939906/3219918/Grishin_RLTP2019_20191107v1.pdf, free. Title from the screen (date of access: 01/08/2023).

5. Nuclotron-based Ion Collider fAcility [Electronic resource]: inf. resource // [site]. - Ed. Jun 01 2023 - URL: https://nica.jinr.ru/ru/, free. Title from the screen (date of access: 01/08/2023).

6. *Paulo Sergio Ramirez Diniz, Eduardo A. B. Da Silva, Sergio L. Netto (2002).* Digital Signal Processing: System Analysis and Design. *Cambridge University Press.* ISBN 0-521-78175-2.

7. Three Stages of The NICA Accelerator Complex Nuclotron-based Ion Col-lider fAcility / V. D. Kekelidze [et al.] // 3rd Large Hadron Collider Physics Conference. — Gatchina : Kurchatov Institute, 2016. — P. 565– 569.