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Fine structure of alpha-decay in actinides

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Abstract

In this work we analyze the role of angular momentum dependence of spectroscopic factors on the fine structure of alpha-decay of actinides. To calculate the width of α-decay the two-potential approach to the tunneling problem has been used. The potential energy for the alpha-cluster system and the wave function of the metastable state has been calculated using dinuclear system model for different values of the octupole deformation of the daughter nuclei. Calculated has been performed for ^{226}Ra, ^{230}Th, and ^{240}Pu.

I. INTRODUCTION

One of the most important decay modes of the actinides is the α-decay. The standard approach to the problem of α-decay consist in the assumption that with certain probability, α-particle can be formed on the surface of parent nucleus. Due to the interplay between Coulomb repulsive and nuclear attractive potentials α-particle is locked in the potential pocket corresponding to a touching configuration with the heavy residual nucleus. Since the potential barrier has finite height, the system can eventually decay, i.e. the α-particle will be emitted.

The probability of decay is governed by the two factors: the preformation factor and the penetrability of the outer barrier. The preformation factor or spectroscopic factor determines the probability with which α-particle can be formed on the surface of heavy nucleus. This quantity completely determined by the structure of the nucleus. The penetrability of the barrier depends strongly on deformation of the daughter nucleus. With deformation increase, Coulomb repulsion becomes weaker and the barrier separating inner and outer regions becomes smaller. Therefore, deformation effects strongly influences the penetrability of the barrier and as such the lifetime of the metastable state corresponding to the molecular system with an α-particle moving on the surface of the heavy nucleus.

The deformation is also important in order to describe the fine structure in alpha-decay. The decay from the ground state of parent nucleus can happens not only to the ground state of daughter nucleus but also to its excited states. The first guess why it can happens is because the potential barrier which alpha-particle has to overcome is not spherically-symmetric and therefore angular momentum of alpha-particle and of the daughter nucleus can change during the tunneling process. Thus, one can try to describe the fine structure of alpha-decay by completely neglecting the dependence of the spectrofactor on angular momentum and taking into account only the deformation effects. However, this approach proved to be unsuccessful. In [1] it was shown that the dependence of spectroscopic factor on angular momentum has to be taken into account as well. The observed correlation between the hindrance factors for the alpha-decay to the 1^- state [2] and the energy of this states also leads to the same conclusion. In this work, we will calculate the angular momentum dependence of spectroscopic factors and analyze to what extend this dependence is responsible for the description of the fine structure of alpha-decay.

II. MODEL

A. Two-potential approach

In order to obtain the expression for the alpha-decay width we use the two-potential approach (TPA) proposed in [3, 4]. In this approach the problem of escape of the system from the metastable state was essentially simplified by reducing it to two separate problems: a bound state plus a non-resonance scattering state. The potential $V(x)$ with a barrier, which contains a quasistationary state with energy E_0 can be divided into two regions, the innner region, $0 < r < R$, and the outer region, $r > R$, where R is taken inside the barrier region. A one dimensional example is presented on Fig. 1.

One can introduce two potentials: the inner-potential

$$U(\mathbf{r}) = \begin{cases} V(\mathbf{r}), & |\mathbf{r}| \leq R, \\ V(R) = V_0, & |\mathbf{r}| > R \end{cases} \tag{1}$$

and the outer potential

$$\tilde{W}(\mathbf{r}) = \begin{cases} V_0 & |\mathbf{r}| \leq R, \\ V(\mathbf{r}), & |\mathbf{r}| > R. \end{cases} \tag{2}$$

The inner potential contains a bound state Φ_0 representing the bound state of the inner Hamiltonian H_0. It was demonstrated that the energy and the width of the quasistationary state can be obtained from the equation:

$$\tilde{E} = E_0 + \langle \Phi_0 | W | \Phi_0 \rangle + \langle \Phi_0 | W \tilde{G}(E) W | \Phi_0 \rangle, \tag{3}$$

where $W = \tilde{W} - V_0$, and the Green's function $\tilde{G}(E)$ is given by

$$\tilde{G}(E) = G_0(E)(1 + \tilde{W}\tilde{G}(E)) \tag{4}$$

with

$$G_0(E) = \frac{1 - |\Phi_0\rangle\langle\Phi_0|}{E + V_0 - H_0}. \tag{5}$$

The width of the quasistationary state is then $\Gamma = -2Im(E)$.

In the following we will apply the expression (3,4,5) to derive the probability for alpha-decay to different states of daughter nuclei (fine structure).

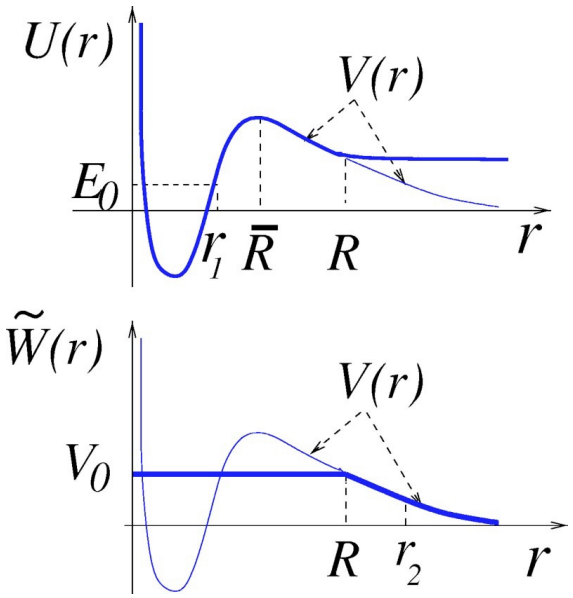

FIG. 1: The inner (U, top) and the outer ($\tilde{W}$, bottom) parts of the potential V defined as in Eqs. (1) and (2), respectively. The separation radius R is chosen well inside the barrier. The energy of the metastable state is E_0. The classical turning points are denoted as $r_{1,2}$.

B. Potential energy

The system of interest consist of heavy fragment and α-cluster close to its surface. There exist several methods to calculate the potential energy of such a system. All different methods though exhibit a general features. Due to the interplay between short-ranger nuclear attraction and long-rage Coulomb repulsion, interaction energy reaches its local minimum at the distances close to the touching configuration of fragments at any value of the angle ϵ (see Fig. 2). As a function of ϵ, potential energy is symmetric function and has a form of two minima at $\epsilon = 0$ and $\epsilon = \pi$ separated by the barrier at $\epsilon = \pi/2$. The height of the barrier is increasing with deformation of the fragment.

In this work we adopt a method proposed in dinuclear system model [5]. The potential is calculated as a sum of Coulomb and nuclear potential. The Coulomb interaction potential is calculated as:

$$U_c = \int \frac{\rho_1(\overrightarrow{r_1})\rho_2(\overrightarrow{r_2})}{|\overrightarrow{r_1} - \overrightarrow{r_2}|} d\overrightarrow{r_1} d\overrightarrow{r_2}, \tag{6}$$

where $\rho_i(\overrightarrow{r})$ $(i = 1, 2)$ are the densities of the heavy nucleus and alpha-particle, respectively.

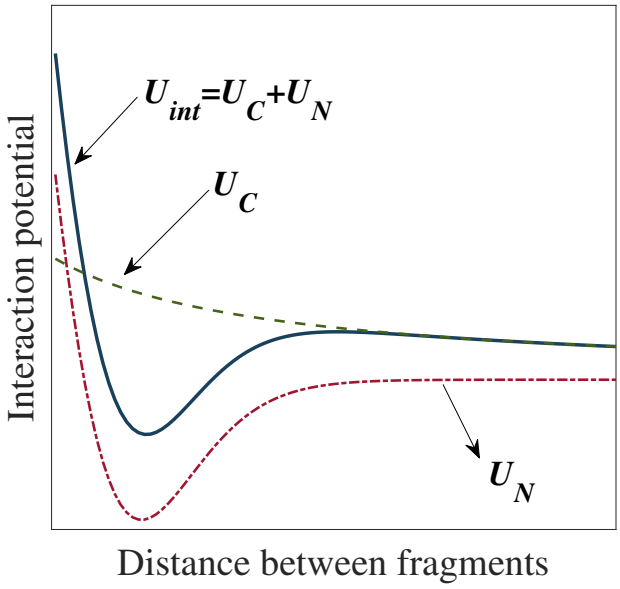

FIG. 2: (Schematic representation of the interaction potential between the fragments of molecule as a function of a relative distance R.

Using the angular momentum algebra [6] one can simplify the expression (6):

$$U_c = \sum_{l_1,l_2}(-1)^{l_1}\sqrt{\frac{4\pi}{2l+1}}\sqrt{\frac{4\pi(2l)!}{(2l_1+1)!(2l_2+1)!}}\frac{\alpha_1\alpha_2}{R^{l+1}}\sum_{m=min(l_1l_2)}^{min(l_1l_2)} C^{l_0}_{l_1ml_2-m}Y_{l_1m}(\Omega_1)Y^*_{l_2m}(\Omega_2) \quad (7)$$

Here,

$$\alpha_1 = \sqrt{\frac{4\pi}{2l_1+1}}Q^{(1)}_{l_1}\rho_1$$
$$\alpha_2 = \sqrt{\frac{4\pi}{2l_2+1}}Q^{(2)}_{l_2}\rho_2 \quad (8)$$

are the multipole moments of the fragments.

The nucleon-nucleon interaction can be represented as:

$$F(\overrightarrow{r_1}-\overrightarrow{r_2}) = C_0\left(F_{in}\frac{\rho_0(\overrightarrow{r_1})}{\rho_{00}} + F_{ex}\left(1-\frac{\rho_0(\overrightarrow{r_1})}{\rho_{00}}\right)\right)\delta(\overrightarrow{r_1}-\overrightarrow{r_2})$$

Here $\rho_0(\overrightarrow{r_1})$ is the density of nucleons, ρ_{00} is the density of nucleons in the center of the nucleus. In the used model, the mother cell core is represented as a composite system of a child core and α particles. Accordingly, the density of the nucleus can be written in terms of the sum of the densities of the components:

$$\rho_0(\overrightarrow{r}) = \rho_1(\overrightarrow{r}) + \rho_2(\overrightarrow{r})$$

Nuclear potential:

$$U_N = \int \rho_1(\overrightarrow{r_1})\rho_2(\overrightarrow{R} - \overrightarrow{r_2})F(\overrightarrow{r_1} - \overrightarrow{r_2})d\overrightarrow{r_1}\overrightarrow{r_2}$$

$$U_N(R) = C_0\frac{F_{in} - F_{ex}}{\rho_{00}}\left(\int \rho_1^2(\overrightarrow{r})\rho_2(\overrightarrow{R} - \overrightarrow{r})d\overrightarrow{r} + \int \rho_1(\overrightarrow{r})\rho_2^2(\overrightarrow{R} - \overrightarrow{r})d\overrightarrow{r}\right) +$$

$$+C_0F_{ex}\int \rho_1(\overrightarrow{r})\rho_2^2(\overrightarrow{R} - \overrightarrow{r})d\overrightarrow{r}$$

Now we can approximate this function with Legendre polynomials. Let's do it up to the third order. The final type of potential:

$$U_N = C_0\sqrt{\frac{1}{2}}P_0(\cos\epsilon) + C_2\sqrt{\frac{5}{2}}P_2(\cos\epsilon) + C_3\sqrt{\frac{7}{2}}P_3(\cos\epsilon). \quad (9)$$

As a result, combining the Coulomb and nuclear potentials and taking into account the terms up to the third order of smallness, we get an approximation of the nuclear potential, which can be used for further calculations. The results of the calculations of the potential energy for the system ^{222}Rn+^{4}He $\rightarrow^{226}$Ra are shown on Fig. 8.

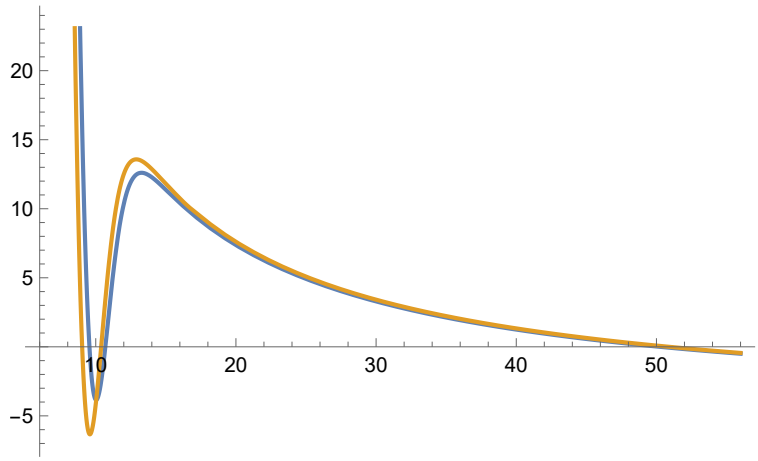

FIG. 3: Potential barrier for the α-cluster dinuclear system ^{226}Ra $\rightarrow^{222}$Rn+^{4}He obtained by numerical calculation (blue line) and by fitting with use of Eq. 9 (yellow).

C. Wave function of the bound state

The wave functions of the yrast states of actinides is taken as a superposition of the shell model configuration and alpha-cluster system [7]

$$\Psi(L) = \cos\gamma_L |SM\rangle_L + \sin\gamma_L |\alpha - cluster\rangle_L. \tag{10}$$

The $|SM\rangle_L$ and $|\alpha - cluster\rangle_L$ are assumed to be orthogonal. The weight of the cluster component in the total wave function is determined as

$$\omega_L = \sin\gamma_L^2. \tag{11}$$

To calculate the weight of α-cluster system, we must solve the Schrodinger equation in mass-asymmetry coordinate. If the cluster system consists of the heavy fragment of mass A_1 and the light fragment of mass $A_2 = A - A_1$, the mass asymmetry coordinate is defined as

$$\xi = \frac{2A_2}{A}, \tag{12}$$

where A is the nuclear mass. The Hamiltonian describing the motion in ξ has the form

$$\hat{H}(I) = -\frac{\hbar^2}{2B}\frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} + U(\xi, I), \tag{13}$$

where B is the effective mass parameter and $U(\xi, I)$ is the potential energy in mass asymmetry for a given angular momentum $I = 0, 2, 4....$ The members of the ground state band are described as the lowest eigenfunctions of $\hat{H}(I)$.

The mass asymmetry ξ is treated as continuous variable. The potential energy is taken in the form

$$U(\xi, I) = a_m(I) + a_2(I)\xi^2 + a_6(I)\xi^6. \tag{14}$$

The coefficients a_2 and a_6 are fitted to describe the calculated energies of the DNS with an alpha-particle and Li as a light clusters

$$U(\xi_i, I) = B_1(\xi_i) + B_2(\xi_i) - B + V(R = R_m, \xi_i, I) + \frac{\hbar\omega_R(\xi_i)}{2}, \qquad (\xi_i = \xi_\alpha, \xi_{Li}). \tag{15}$$

Here, B_1, B_2, and B are the binding energies of the fragments and the compound nucleus, respectively. The experimental ground–state masses are used in the calculations. Shell

effects and pairing correlations are included in the binding energies. The nucleus–nucleus potential in (15)

$$V(R,\xi,I) = V_{Coul}(R,\xi) + V_{nucl}(R,\xi) + V_{rot}(R,\xi,I) \tag{16}$$

is the sum of the Coulomb potential, nuclear interaction potential, and the centrifugal energy. The details of calculations are presented in [5]. The nucleus–nucleus potential $V(R,\xi,I)$ is calculated in touching configuration of the fragments that corresponds to the minimum of the potential pocket in relative distance coordinate $R = R_m$. The last term in (15) represents the energy of zero point vibration in this pocket.

The weight of the cluster component in the total wave function can be determined from the wave functions in mass asymmetry obtained above as

$$\omega_L = \sin\gamma_L^2 = \int_{\xi_\alpha/2}^{\infty} |\Psi_I(\xi)|^2 \xi d\xi. \tag{17}$$

For further calculations we need a wave function of α-cluster dinuclear system $|\alpha - cluster\rangle_L \equiv \phi_\alpha$. Lets assume that molecular system corresponding to the compound nucleus (A,Z) consists of an axially-symmetric quadrupole-deformed fragment (A_1,Z_1) and spherical fragment (A_2,Z_2). The deformation of the fragment (A_1,Z_1) is defined by the parameter $\beta_{20} \equiv \beta$. The distance between the centers of the fragments will be denoted as R. The orientation of the molecule with respect to the laboratory system can be described by the angles $\Omega_R = (\phi_R,\theta_R,0)$, which define the orientation of the vector $\mathbf{R}$. The orientation of the first fragment can be described by the angles $\Omega_H = (\phi_H,\theta_H,0)$ which define the orientation of its symmetry axis. Both sets of Euler angles Ω_R and Ω_H are defined with respect to the laboratory system (see Fig. 4). The relative orientation of fragments can also be described by angle ϵ, which is the plain angle between the vector $\mathbf{R}$ and the symmetry axis of the deformed fragment.

The classical expression for the kinetic energy is

$$T = \frac{1}{2}\int \rho(\mathbf{r})\dot{\mathbf{r}}^2 d\mathbf{r}, \tag{18}$$

where $\rho(\mathbf{r})$ is the density of the molecule. Assuming weak overlap of the fragments:

$$\rho(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{r}), \tag{19}$$

where $\rho_i(\mathbf{r})$, $(i = 1, 2)$ are the densities of the fragments. Therefore, the kinetic energy can be rewritten as

$$T = \frac{1}{2}\int \rho_1(\mathbf{r})\dot{\mathbf{r}}^2 d\mathbf{r} + \frac{1}{2}\int \rho_2(\mathbf{r})\dot{\mathbf{r}}^2 d\mathbf{r}. \tag{20}$$

Let us, in addition to the laboratory system, introduce the coordinate systems related to the centers of mass of each fragments. The vectors $\mathbf{R_1}$ and $\mathbf{R_2}$ describe the position of these coordinate systems with respect to the laboratory system. Assuming that the center of laboratory system coincide with the center of mass of the molecule, we have:

$$\begin{aligned} \mathbf{R_1} &= \frac{A_2}{A}\mathbf{R}, \\ \mathbf{R_2} &= -\frac{A_1}{A}\mathbf{R}. \end{aligned} \tag{21}$$

Then we can write kinetic energy as

$$T = \frac{1}{2}\int \rho_1(\mathbf{r})(\dot{\mathbf{r}}_\mathbf{1} + \dot{\mathbf{R}}_\mathbf{1})^2 d\mathbf{r_1} + \frac{1}{2}\int \rho_1(\mathbf{r})(\dot{\mathbf{r}}_\mathbf{2} + \dot{\mathbf{R}}_\mathbf{2})^2 d\mathbf{r_2}. \tag{22}$$

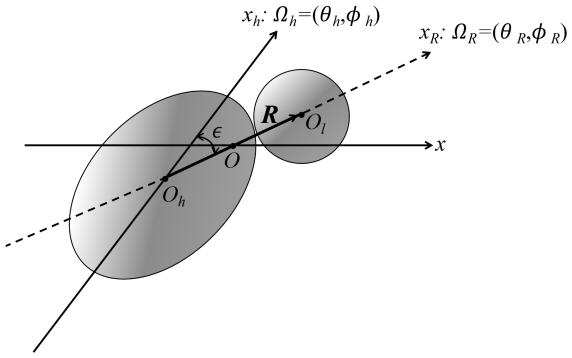

FIG. 4: (Colour online) Schematic representation of the molecular system with indication of the degrees of freedom used. Orientation of the vector $\mathbf{R}$, connecting the centers of the nuclei, with respect to the laboratory system Ox is defined by the angles $\Omega_R = (\theta_R, \theta_R)$. Orientation of the intrinsic coordinate system of the quadrupole-deformed fragment $O_h x_h$ with respect to the laboratory system is described by the angles $\Omega_h = (\theta_h, \phi_h)$. Angle ϵ is the plane angle between $\mathbf{R}$ and the symmetry axis of the deformed fragment.

Taking into account that the mass dipole moments of the fragment are equal to zero, that is

$$\int \rho_i(\mathbf{r})\mathbf{r_i} d\mathbf{r_i} = 0, \tag{23}$$

and neglecting the excitations of the spherical fragment (A_2, Z_2), we can write:

$$\begin{aligned} T &= \frac{1}{2}\int \rho_1(\mathbf{r_1})\dot{\mathbf{r}}_\mathbf{1}^\mathbf{2}\mathbf{dr_1} + \frac{\mathbf{1}}{\mathbf{2}}\mu\dot{\mathbf{R}}^\mathbf{2} \\ &= \frac{1}{2}\int \rho_1(\mathbf{r_1})\dot{\mathbf{r}}_\mathbf{1}^\mathbf{2}d\mathbf{r_1} + \frac{1}{2}\mu\dot{R}^2 + \frac{1}{2}\mu R_m^2(\dot{\theta}_R^2 + \sin^2\theta_R\dot{\phi}_R^2), \end{aligned} \tag{24}$$

where $\mu = m_0\frac{A_1A_2}{A}$ is the reduced mass of the molecule and m_0 is the nucleon mass.

Evaluating integral in (24), we obtain

$$T = \frac{1}{2}\Im_H(\dot{\theta}_H^2 + \sin^2\theta_H\dot{\phi}_H^2) + \frac{1}{2}\mu\dot{R}^2 + \frac{1}{2}\mu R_m^2(\dot{\theta}_R^2 + \sin^2\theta_R\dot{\phi}_R^2), \tag{25}$$

where $\Im_H$ is the moment of inertia of the fragment (A_1, Z_1).

After the quantization, the kinetic energy takes the following form:

$$T = -\frac{\hbar^2}{2\mu R^2}\frac{\partial}{\partial R}R^2\frac{\partial}{\partial R} + \frac{\hbar^2}{2\mu R_m^2(\epsilon)}L_R^2 + \frac{\hbar^2}{2\Im_h(\xi)}L_h^2, \tag{26}$$

where angular momentum operators are defined as:

$$\begin{aligned} &L_i^2 = -\frac{1}{\sin\theta_i}\frac{\partial}{\partial\theta_i}\sin\theta_i\frac{\partial}{\partial\theta_i} - \frac{1}{\sin^2\theta_i}\frac{\partial^2}{\partial\phi_i^2}, \\ &(i = R, h). \end{aligned} \tag{27}$$

Neglecting the tunneling of the system from the pocket in the potential energy, we can compare the characteristic frequency for the vibration in relative distance coordinate R and moments of inertia which describe rotation of the heavy fragment $\Im_H$ and relative rotation $\Im_R \approx m_0A_1A_2/AR_m^2$:

$$\hbar\omega_R \gg \Im_H, \Im_R. \tag{28}$$

That is, we can assume that lowest excitations in the molecular system are related to the motion in the angular degrees of freedom. Zero-point vibrations in coordinate R plays a role of an additional potential energy in the Hamiltonian describing angular vibrations [8].

The α-cluster wave function ϕ_α can be obtained by diganalizing the Hamiltonian with kinetic energy determined in (26) and potential energy taken in the form (9). This Hamiltonian can be diagonalized on the basis set:

$$[Y_{l_1}(\Omega_R) \times Y_{l_2}(\Omega_H)]_{LM}\,. \tag{29}$$

For ground state wave function the wave function can be written as:

$$\phi_\alpha(R, \Omega_H, \Omega_R) = \phi(R)\sum_l a_l\,[Y_l(\Omega_R) \times Y_l(\Omega_H)]_{00} = \phi(R)\sum_l \tilde{a}_l P_l(\cos\epsilon). \tag{30}$$

Taking into account that the cluster component presents only a part of total wave function of the bound state, the preformation factor of the alpha-decay with daughter nucleus in the state with angular momentum l can be written as:

$$S_l^2 = \omega_0 |\tilde{a}_l|^2. \tag{31}$$

D. Width of α-decay

In order to calculate the width of α-decay it is necessary to caculate the imaginary part of the Eq. 3. The first two terms are real, that is, it is necessary to find the third. Let's first consider the case of spherically symmetric potential (daughter nucleus is spherical). To do this, we should evaluate the Green's function:

$$G = -\frac{2\mu}{2\pi\hbar^2} \int dk \frac{\Psi_{kl}(r,\Omega)\Psi^*_{kl}(r',\Omega')}{k_0^2 - k^2 - i\gamma}$$

, where

$$\Psi_{kl} = \frac{\chi_{kl}(r)}{r}(Y_l \cdot Y_l) = \sqrt{\frac{2l+1}{4\pi}}\frac{\chi_{kl}(r)}{r}P_l(cos\epsilon)$$

The function Ψ_{kl} is a wave function for an undeformed (Coulomb only) potential $W(r)$.

$$< \Phi_0 | W G_W W | \Phi_0 >= -\frac{2\mu}{\hbar^2}\frac{1}{2\pi}\sum_l \int \frac{dk}{k_0^2 - k^2 - i\gamma} \left| \int d^3 r \Psi_0(r,\epsilon) W \Psi_{kl}(r,\psi) \right|^2$$

In order to calculate the integral

$$M = \int d^3 r \Psi_0(r,\epsilon) W(r,\epsilon) \Psi_{kl}(r,\epsilon)$$

we use Gauss-Ostrogradsky's theorem which yields:

$$M = \frac{\hbar^2}{2\mu} \int d(cos\epsilon) \left(\phi_0(r,\epsilon)\nabla\chi_{kl}(r,\epsilon) - \chi_{kl}(\epsilon)\nabla\phi_0(r,\epsilon)\right)$$

$$\phi_0(r,\epsilon) = \sum_{l'} S_{l'}\phi_{l'}(r)P_{l'}(cos\epsilon)$$

Then:

$$M = \frac{\hbar^2}{2\mu}\frac{2l+1}{4\pi}\sum_{l'} S_{l'} \int d(\cos\epsilon) \left(\phi_{l'}(r)P_{l'}(\cos\epsilon)\frac{\partial}{\partial r}\chi_{kl}(r)P_l(\cos\epsilon) - \chi_{kl}(r)P_l(\cos\epsilon)\frac{\partial}{\partial r}\phi_{l'}(r)P_{l'}(\cos\epsilon)\right)$$

Using the orthogonality relations of Legendre polynomials, we obtain:

$$M = \frac{\hbar^2}{2\mu}\frac{2l+1}{4\pi}\sum_l S_l\left(\phi_l(r)\frac{\partial}{\partial r}\chi_{kl}(r) - \chi_{kl}(r)\frac{\partial}{\partial r}\phi_l(r)\right)$$

And the width of the decay:

$$\Gamma = \sum_l \left(\frac{2l+1}{4\pi}\right)^2 \frac{\hbar^2}{4\mu k_0} S_l^2 \left(\phi_0(r)\frac{\partial}{\partial r}\chi_{kl}(r) - \chi_{kl}(r)\frac{\partial}{\partial r}\phi_0(r)\right)^2 \tag{32}$$

This solution is obtained under the assumption that the potential is spherically symmetric. We depedence of the potential on deformation can be taken into account as perturbation:

$$W(r,\epsilon) = \frac{W_0}{r} + \frac{W_2}{r^3}P_2(cos\epsilon) + \frac{W_3}{r^4}P_3(cos\epsilon) = \frac{W_0}{r} + \bar{W}(r,\epsilon)$$

The function $\chi(r,\psi)$ we will find in the first approximation of the perturbation theory:

$$\chi(r,\epsilon) = \chi^{(0)}(r,\epsilon) + \sum_{l'}\int dk' \frac{<\chi_{kl}|\bar{W}|\chi_{k'l'}>}{E_{kl} - E_{k'l'}}\chi^{(0)}_{k'l'}$$

Taking into account the corrections to the wave function, we obtain for the width of alpha-decay:

$$\Gamma = \sum_l \Gamma_l$$

$$\Gamma_l = \left(\frac{2l+1}{4\pi}\right)^2 S_l^2 j_{kl}^2 + 2\sum_{k'l'}\frac{(2l+1)(2l'+1)}{(4\pi)^2}\int dk \frac{g_2+g_3}{E_{kl}-E_{k'l'}} S_l S_{l'} j_{kl} j_{k'l'}, \tag{33}$$

where

$$j_{kl} = \phi_0\frac{\partial}{\partial r}\chi^{(0)}_{kl} - \chi^{(0)}_{kl}\frac{\partial}{\partial r}\psi_0$$

As already indicated, the first term is the width obtained neglecting deformation dependence of the spherical potential, and the second term is the correction.

III. RESULTS

Calculations were performed for three cores - ^{226}Ra, ^{230}Th, ^{240}Pu. The nuclei have both quadrupole and octupole deformations. While the quadrupole deformation can be obtained from the calclations [9], the octupole deformation appears mainly because of the softness

of the nucleus and polarizing effect of the alpha-cluster. In order to estimate the effect of octupole deformation, the calculations have been performed for different values of β_3. In the present version of the calculations, we neglect the second term in the expression (33) and only study the fine structure due to the angular momentum dependence of the preformation factor.

On the Figs. **??**, the results presented as a black dots indicate the values obtained in the absence of octupole deformation. The red lines are experimental values.

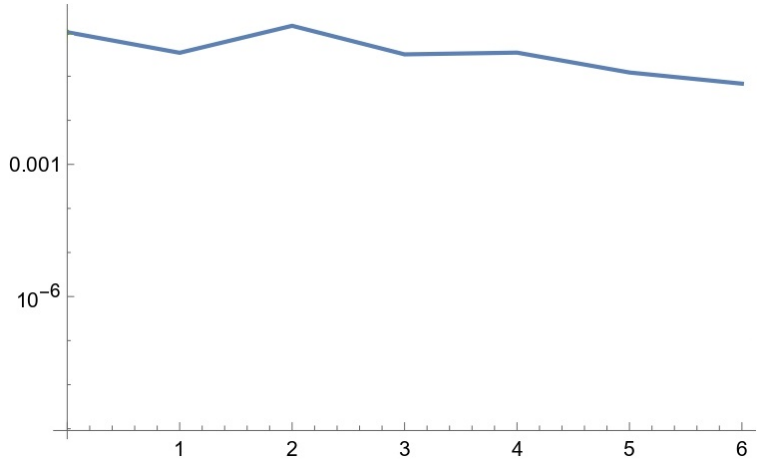

FIG. 5: Dependence of the intensity ratios on the orbital moment for ^{230}Th with S_l=1.

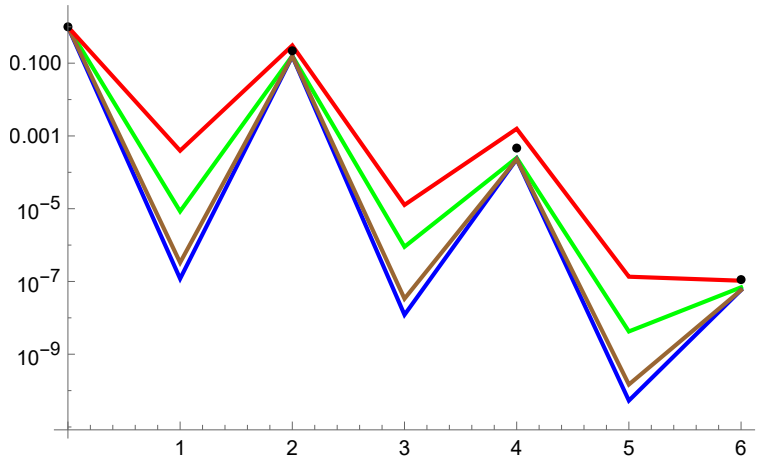

FIG. 6: Dependence of the intensity ratios on the orbital moment for ^{230}Th.

Let's compare the results without taking into account the spectrfactors (they are assumed to be equal to 1) with the case, when we take them into account. As can be seen on the example of ^{230}Th, dots lie on a line closer to a straight line without spectrfactors and that

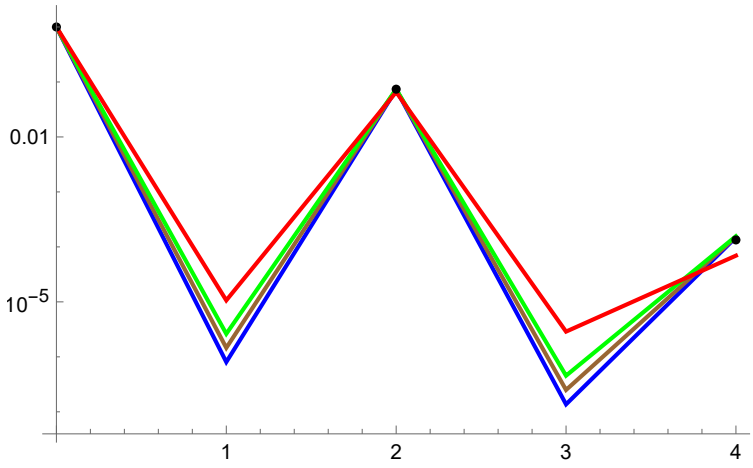

FIG. 7: Dependence of the intensity ratios on the orbital moment for ^{226}Ra.

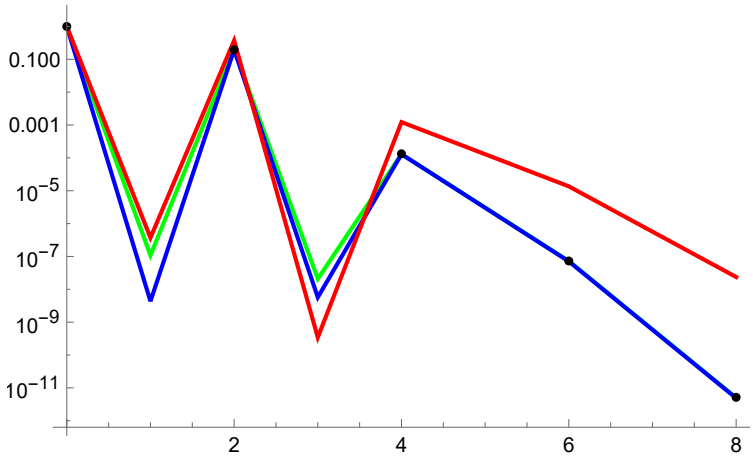

FIG. 8: Dependence of the intensity ratios on the orbital moment for ^{240}Pu.

it is their presence that affects the shape of the curves. Due to this, the probability of transition to the states with odd angular momentum I is reduced.

We also note that the of octopole deformation weakly affects the intensity of transitions to the states with even I. An increase in the deformationleads to an increase of the intensity ratio.

The discrepancy with the experimental results is due, firstly, to the lack of experimental data for the octopole deformation parameter β_3 for most nuclei. The second reason may be that the second term in the Eq. (33) was not taken into account. This term should act to equalize the transition rates to the states of daughter nucleus with different angular momenta.

[1] W. M. Seif, A. Adel, N. V. Antonenko, and G. G. Adamian, Phys. Rev. C **107**, 044601 (2023).

[2] R.K.Sheline, M.A.Riley, Phys.Rev. C **61**, 057301 (2000).

[3] S. A. Gurvitz, Phys. Rev. A **38**, 1747 (1988).

[4] S. A. Gurvitz, P. B. Semmes, W. Nazarewicz,and T. Vertse, Phys. Rev. A **69**, 042705 (2004).

[5] G.G. Adamian, N.V. Antonenko, R.V. Jolos, S.P. Ivanova, and O. Melnikova, Int. J. Mod. Phys. E **5**, 191 (1996).

[6] D. A. Varsalovich, A.N. Moskalev, V.K. Khersonsky, *Quantum theory of angular momentum*, Nauka, Leningrad, 1975.

[7] T.M.Shneidman, G.G.Adamian, N.V.Antonenko, R.V.Jolos, S.-G.Zhou, Phys.Rev. C **92**, 034302 (2015)

[8] A.B. Migdal, V.P. Krainov, *Approximate methods of quantum mechanics*, Nauka, Moscow, 1966.

[9] P. Möller, A.J. Sierk, T. Ichikawa, and H. Sagawa,Nucl. Data Tables **109-110**, 1-204 (1995).