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Neutrino propagator validation for Neutrino Telescope Simulation

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Neutrino propagator validation for Neutrino Telescope Simulation

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Abstract

This report focuses on the process of neutrino-interaction events modeling in NTsim - a framework that consists of different modules for neutrino telescopes simulation. Work, described in this paper, is about the simulation for one specific experiment - Baikal-GVD. Baikal neutrino telescope is a deep water installation, which purpose is to detect high energy astrophysical neutrinos with energies up to 10^{18} eV. The NuPropagaror module's aim is to propagate neutrinos through the medium in a proper way so the process of secondary particle appearing is correct and has no lack of physical data.

Simulated data matching with theoretical predictions provides the simulation veracity. Thus, the main goal of the work is to create algorithms for NuPropogator's verification with the analyses of Bjorken variables, which are usually used to describe deep inelastic interaction process.

Introduction

NTSim framework is a project, which goal is to provide scientists with an opportunity to simulate data for neutrino experiments. The framework consists of a wide number of modules, where each module has its own specific range of capacities for the requested task.

The part of work, which this report concerns, is based on NuPropagator module testing. The module propagates neutrino through the matter, counting data for all physical processes included into the event. To code this process properly it is important to understand the physics of neutrino interactions. Neutrinos interact with nuclei of the detection medium through exchange of a charged vector boson W^+, i.e. charged current (CC) interactions or through exchange of the neutral vector boson Z_0, i.e. neutral current (NC) deep inelastic scattering interactions [1]. The process of lepton interaction is described using a parton model, where LHAPDF, a standard tool for evaluating parton distribution functions (PDFs) in high-energy physics, is used in the propagation procedure. In the parton theory deep inelastic scattering is described with Bjorken varibles, which contain the whole information about the procedure. Distributions of the main simulated variables, such as $x_{bjorken}$ and $y_{bjorken}$, have been received and analyzed in order to check the simulation process.

This work helped me to improve my using interpreted object-oriented programming language Python3. The language was chosen considering its user-friendly appliance and its wide application in NTSim. Also, skills of using different data libraries such as LHAPDF and Vegas Python were acquired.

Chapter 1

Neutrino interactions

1.1 Bjorken scaling

As was mentioned previously, neutrinos interact with nuclei of the medium by either the exchange of a charged vector boson W^+, i.e. charged current (CC) interactions or by the exchange of the neutral vector boson Z_0, i.e. neutral current (NC) deep inelastic scattering interactions. The ratio of the CC to NC interaction cross sections is approximately 2:1. The CC interactions produce charged secondary scattered leptons while the NC interactions produce neutrinos[1]. These interaction channels are illustrated on the picture below.
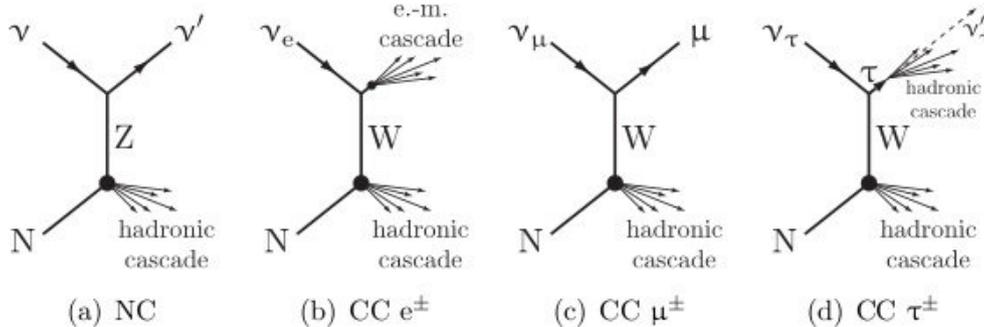

Figure 1.1: NC and CC ν interactions

To describe such processes it is needed to calculate cross section, but firstly substitution of variables needs to be done, so that the process can be described using x, y, Q^2. To determine kinematic variables lepton-nucleus scattering demonstrated in the figure 1.2. can be described.

Firsly, the notation: $K = (E, \overrightarrow{k})-$ four - momentum of initial lepton; $K' = (E, \overrightarrow{k'})-$ four - momentum of scattered lepton; $P = P_N = (M, \overrightarrow{0})-$ four - momentum of target nucleon; $q = K - K'-$ four - momentum transfer; $Q^2 = -q^2 = -(K - K')^2-$ squared four - momentum transfer.

Squared total energy of parton - boson system:

$$E^2 = m^2 c^4 = (x_p P_N + q)^2 = x_p^2 P_N^2 + 2x_p(P_N \cdot q) + q^2 = x_p^2 M^2 + 2x_p(P_N \cdot q) + q^2 = m^2 \quad (1.1)$$
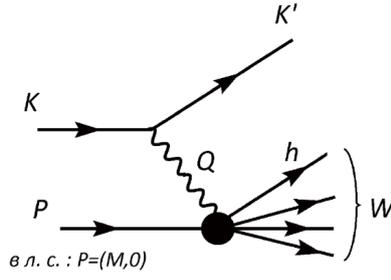
Figure 1.2: Lepton-nucleus scattering.

Where the first summand is for parton four - momentum, which equals the x fraction of nucleon four - momentum, and the second is for the boson four - momentum. If the parton mass is small, we can neglect it in comparison with other kinematic values. Also we can neglect summands with squared nucleon mass, because it will be way more less than 1:

$$2x_p(P_N \cdot q) + q^2 = 0 \Rightarrow q^2 = -2x_p(P_N \cdot q) \tag{1.2}$$

$$x_p = -\frac{q^2}{2(P_N \cdot q)} \tag{1.3}$$

Moreover we know that q is the difference between the initial lepton four - momentum and the scattered lepton four - momentum. So we can substitute for q this difference:

$$q = K - K' \tag{1.4}$$

$$x_p = -\frac{(K - K')^2}{2(P_N \cdot q)} \tag{1.5}$$

We can also get the scalar multiplication of nucleon four - momentum and transferred four momentum:

$$(P_N \cdot q) = (M, \vec{0}) \cdot (E - E', \vec{k} - \vec{k'}) = M(E - E') - \vec{0}(\vec{k} - \vec{k'}) = M(E - E') \tag{1.6}$$

Physical interpretation of x - fractional momentum transfer (the fraction of the initial lepton four - momentum transferred to the nucleon)

$$x_p = x_{Bjorken} = x = -\frac{(K - K')^2}{2M(E - E')} \tag{1.7}$$

For kinematic description of deep inelastic scattering we also can use such values as: y - fractional energy transfer (the fraction of the initial lepton energy transferred to the nucleon) and theta - scattering angle. For y we need to find the lepton's energy loss:

$$\omega = \frac{(q \cdot P_N)}{M} = \frac{M(E - E')}{M} = E - E' \tag{1.8}$$

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So the fractional energy loss will be:

$$y = \frac{\omega}{E} = \frac{E - E'}{E} \tag{1.9}$$

From the squared four - momentum transfer we can find the scattering angle:

$$Q^2 = -q^2 = -(K - K')^2 = 2EE' - 2\overrightarrow{k}\,\overrightarrow{k'} \tag{1.10}$$

$$Q^2 = 2(EE' - |k||k'|cos\Theta) = 2EE'(1 - cos\Theta) \tag{1.11}$$

Final formula for the scattering angle:

$$Q^2 = 2EE'(1 - cos\Theta) \Rightarrow cos\Theta = 1 - \frac{Q^2}{2EE'} = 1 + \frac{(K - K')^2}{2EE'} \tag{1.12}$$

There is one more kinematic variable z for the fractional energy transfer, taken away by hadron:

$$z = \frac{E_h}{\omega} = \frac{E_h}{E - E'} \tag{1.13}$$

Where the E_h is for the hadron energy. As a result of the calculations presented above, the kinematic values for deep inelastic scattering description are determined. Hence, the differential cross-section can be conveniently expressed in terms of two dimensionless variables x and y Bjorken. With this result we can study cross-section by analysis of x, Q^2 and y variables.

$$\frac{d^2\sigma}{dQ^2 d\omega} \rightarrow \frac{d^2\sigma}{dxdy} \tag{1.14}$$

With the Jacobian of the transformation equal $dQ^2 d\omega = 2ME^2 ydxdy$.

1.2   The NTsim framework

NTsim is a framework, which includes a number of different tools for simulation for different neutrino experiments, like IceCube, KM3NET/Arca, Baikal-GVD and P-ONE. The structure of the basic engine can be described as combination of:

- g4camp - module based on Geant4 and geant4pybind

- VIOLINE – visual interface based on Qt5

- Detector – GVDDetector, more options are under development

- primaryGenerators – module, which generates events for such particles as ν, μ, e, γ, π, p, etc.

- primaryPropagators – module, that propagates particles through the matter

The main groups of modules are Propogators and Generators, where the primary one includes such options as:

- CherenkovPropagator - propagates Cherenkov photons

- g4propagator - propagates particles, included in g4camp facilities

- UMpropagator - propogates muons

- nuPropagator - propagates neutrinos

- trackCherenkov - is in charge of Cherenkov photons generation along the secondary particles' tracks

And the following one includes such options as:

- ToyGen - initializes particles, included in g4camp facilities

- nuGenerator - initializes neutrinos

The chain of neutrino event production is presented in the picture 1.3.
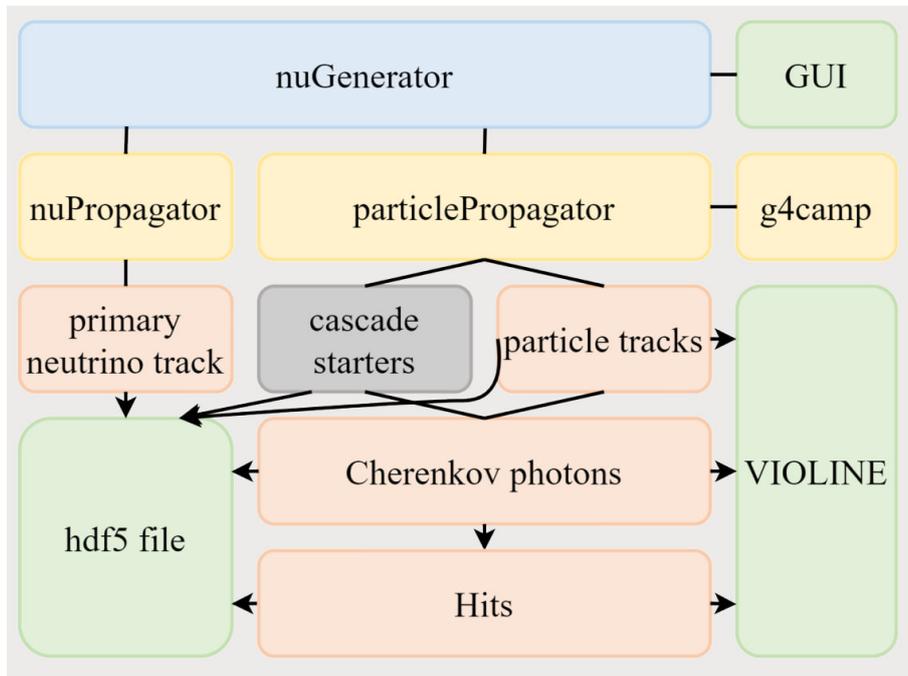
Figure 1.3: Simulation chain

NuPropagator is a software package, which aim is to calculate the fluxes of neutrinos and leptons at an arbitrary point on the Earth. This software package is written in Python using the Numpy and Scipy modules. The program takes into account neutrino flux on the earth's surface and calculates the probability of neutrino passing from one point to another.

As this report only touches upon neutrino interactions, it is important to mention that neutrino events can be produced using both, whole NTsim and separated NuPropagator package. In the practical part the comparison of differently produced events is presented.

Chapter 2

The practical part

2.1 Visualisation of Bjorken kinematic variables

While evaluating the differential cross-section using programming tools, it is important to have cognizance of which parton density functions are operated. In NTSim framework for cross-section calculations the LHAPDFD set named 'CT10nlo' is used. Graphs for these with different $Q^2(GeV^2)$ are presented below:
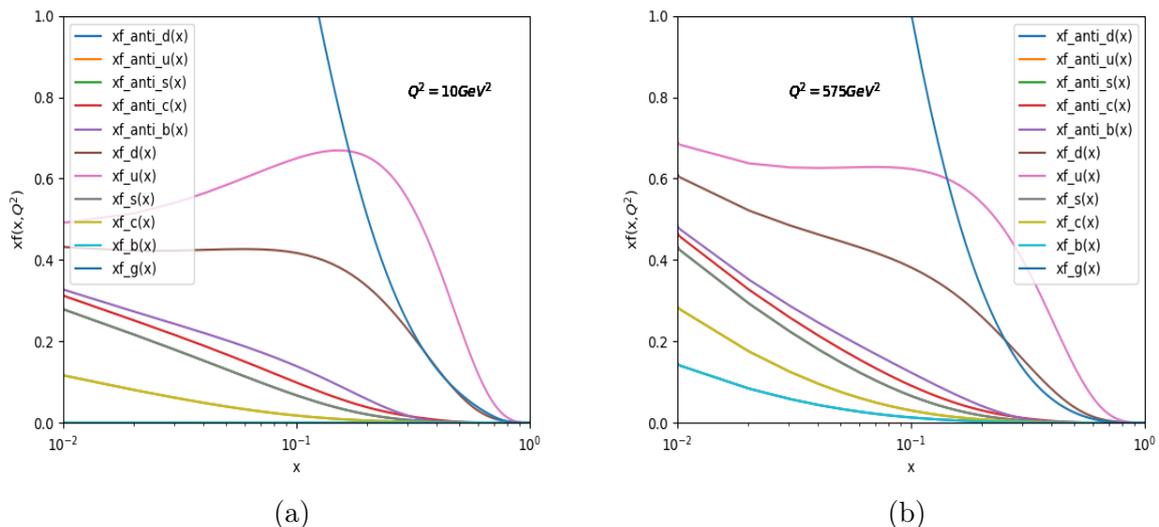

(a)   (b)

Figure 2.1: Parton density distributions for $Q^2 = 10(a), 575(b)Gev^2$

Primary task of the work was to compare data simulated by NuPropagator's functions and simulated by the general framework. The results of this part were not such great, as they were expected, because the distributions were not only uniform with each other, but also had some significant differences with theoretical predictions. The complete difference between these two methods was explained by two different algorithms for choosing x and y values. The NuPropagator returns whole array of paired x and y values, for requested event, while its call in NTSim returns only one pair of all the calculated values, for this reason the kinematic values distributions are not similar.

But the issue with the difference between theoretical values for cross-section and collected distributions for x had no such simple explanation. The data produced by NuPropagator is shown with the orange color in the histograms below and has a sharp peak for x values around zero. And as it is illustrated in the picture 2.3 there should not be such behaviour.

To properly scrutinize the difference, it is important to compare x, which are paired with fixed y values equal to 0.1, 0.5, 0.9, with the theoretical calculations. Consequently, the distributions presented in the figure 2.3. were created. Furthermore, the energy limits were changed in order to find the issues in more valuable statistics. As it can be seen on the figure 2.3, the distributions still have some specific behaviour, which can not be recognized as predicted. The distribution for $E_\nu = 10^3 \text{GeV}$ shows the sharp peak. To explore this discrepancy it have been offered to explore 2D distributions for x and y. These distributions are presented with simulated data in the picture 2.4. and with theoretical calculations in the picture 2.5. .

As it can be seen on the pictures, simulated and calculated data have differences, the causes for this kind of behaviour can be found in several spots. First one is the LHAPDF model, which parton distributions are used for both the calculation and simulation. Testing different models and selection of the most compatible one is to be included in further work perspectives. Another cause can be the integration of cross-sections with Vegas module. The procedure of using this python package is including the determination of integration limits and scale,changing which in oder to find the most suitable one is also a part of upcoming work.
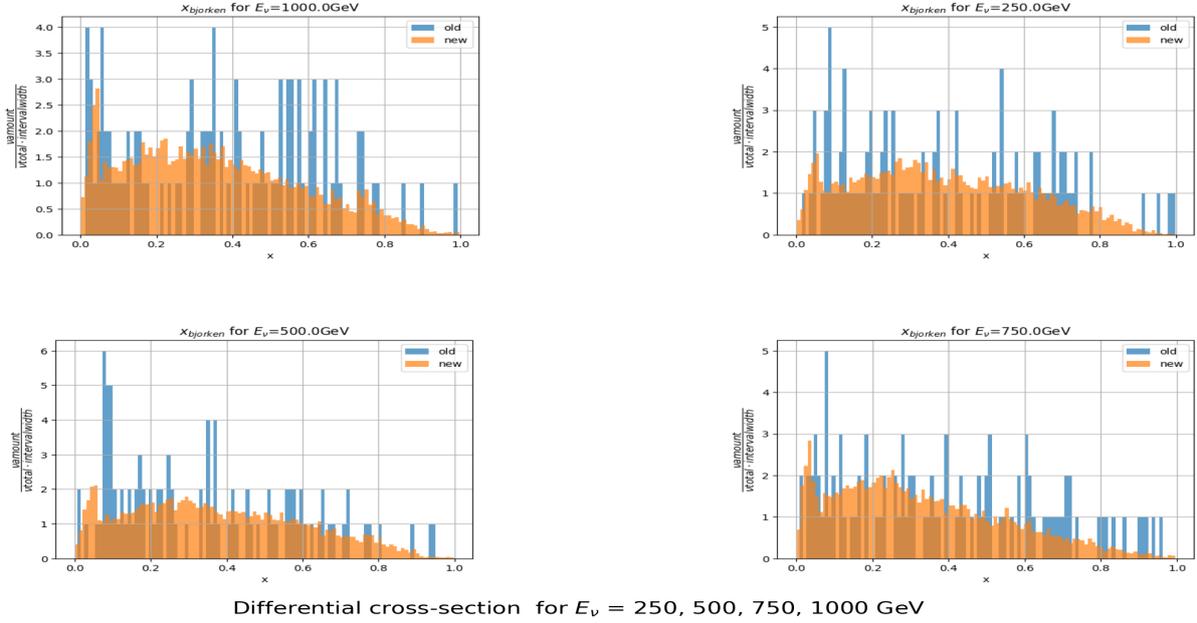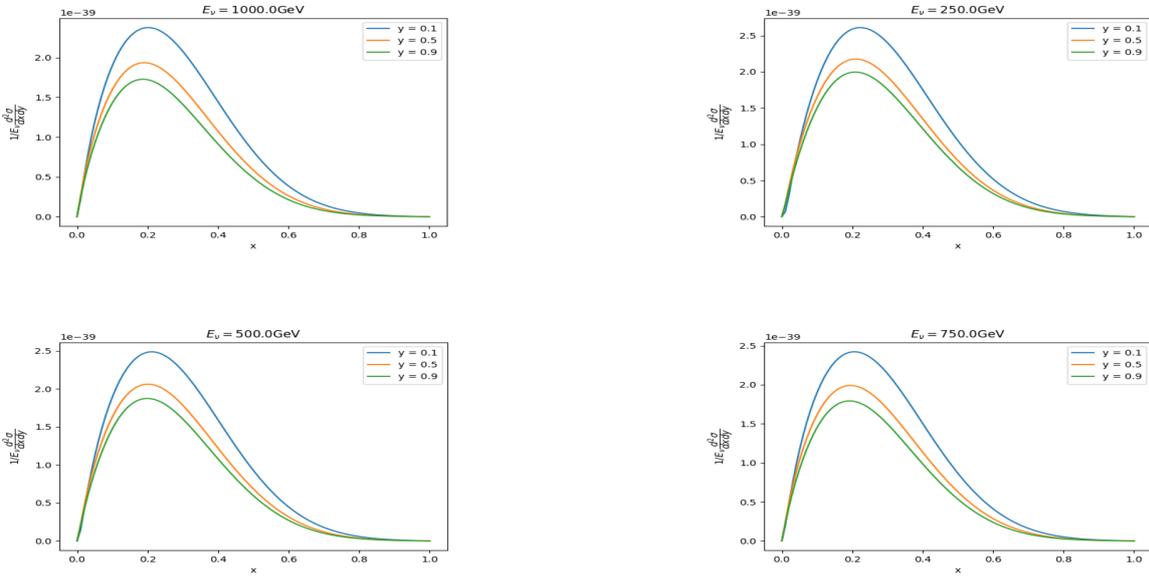
Figure 2.2: x Bjorken distributions(old corresponds to NTsim data and new corresponds to NuPropagator data)(upper picture). Theoretical values for cross - section(bottom).

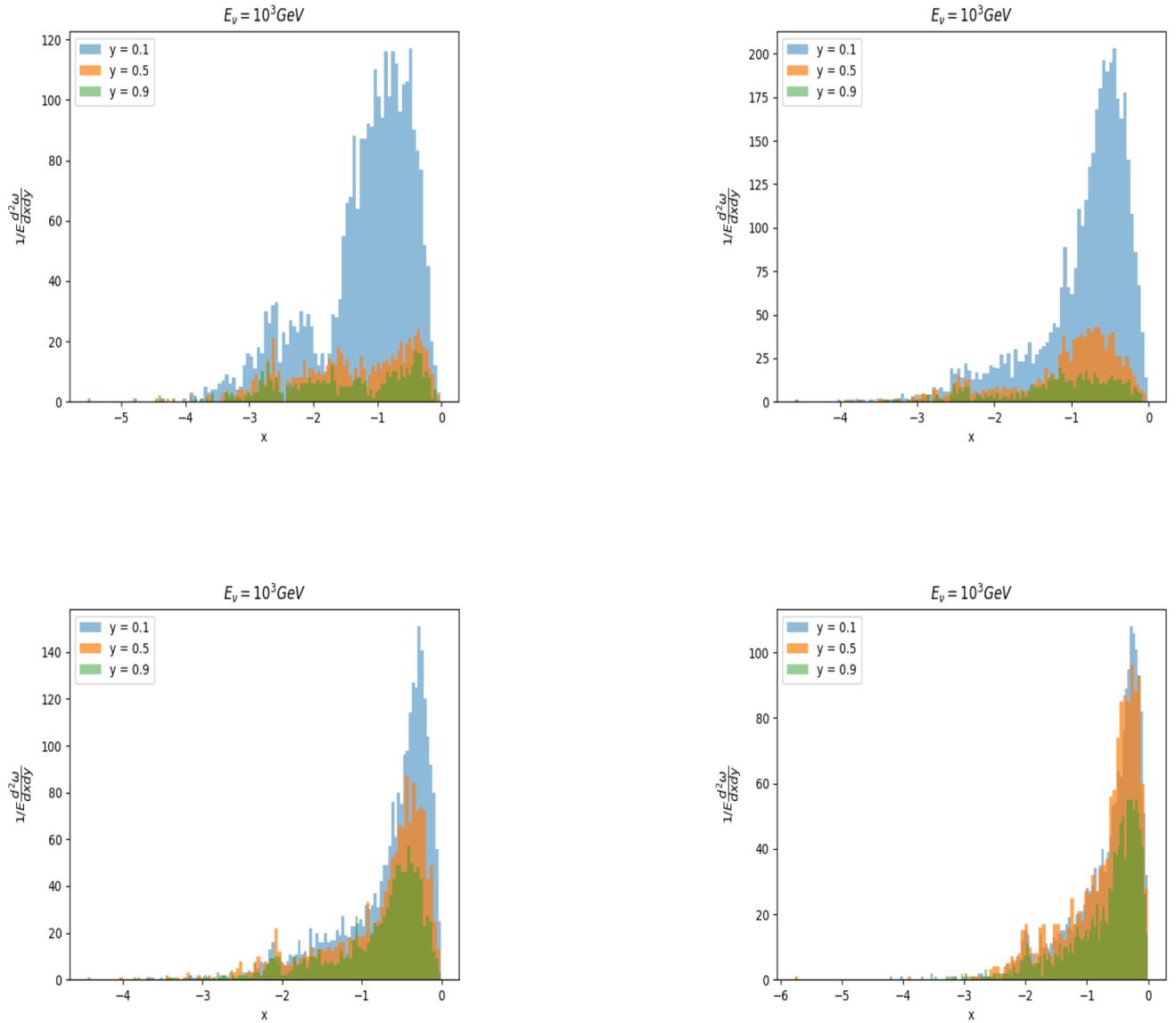Fractional momentum transfer for $E_\nu = 10^3, 10^4, 10^5, 10^6$ GeV

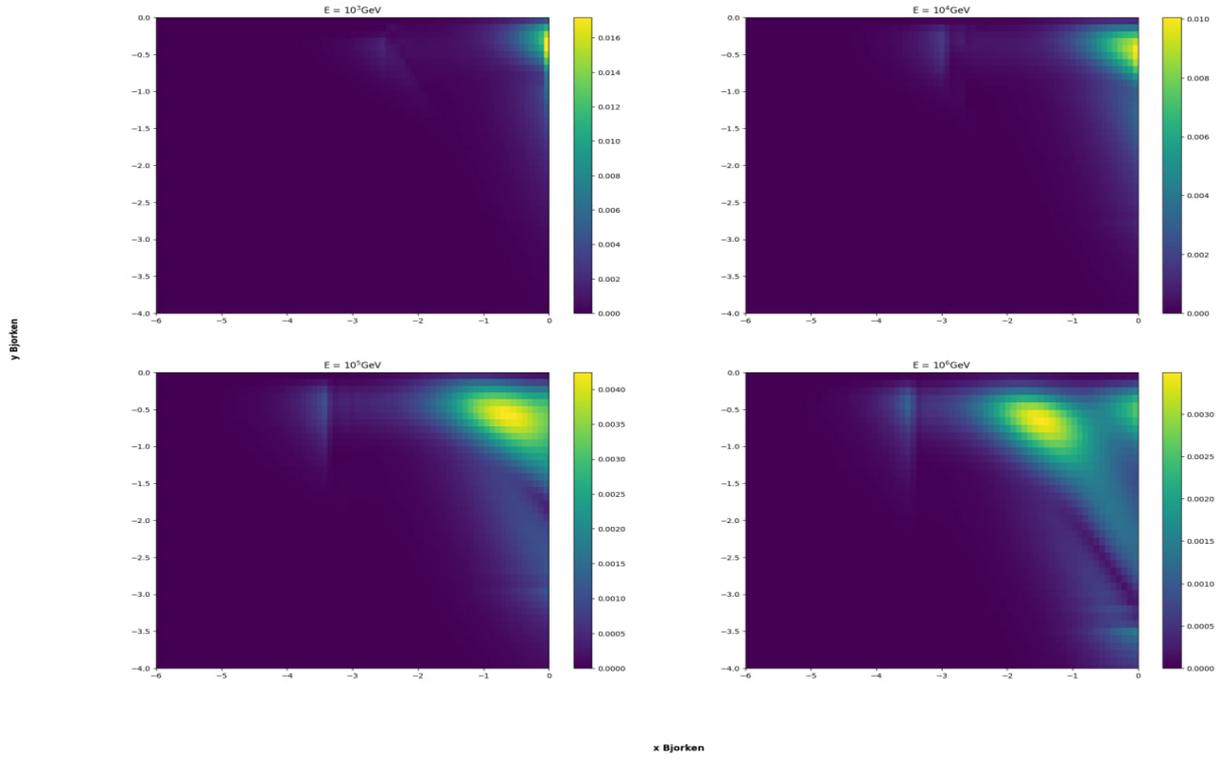Figure 2.3: X Bjorken distributions with fixed y = 0.1, 0.5, 0.9, logarithmic scale.
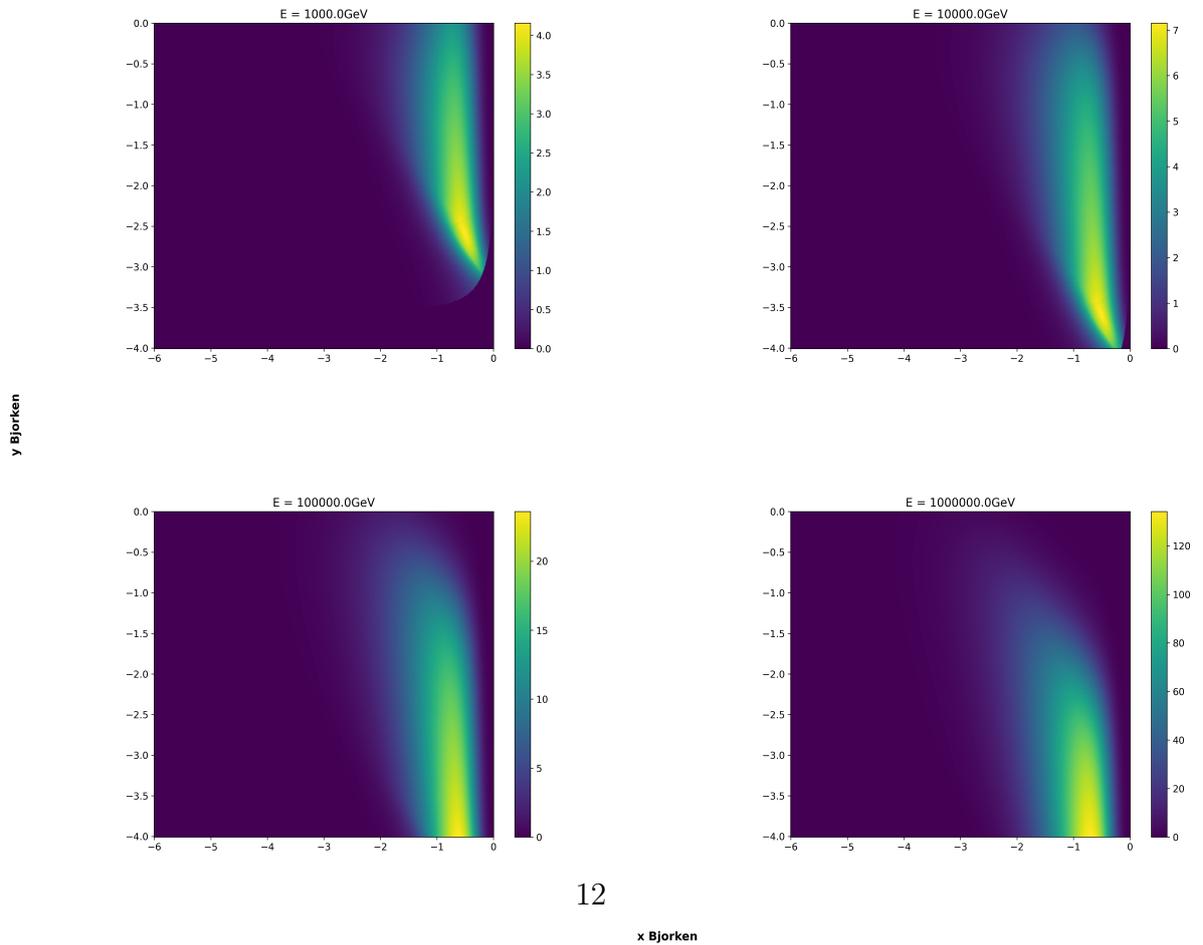
Figure 2.4: X and Y distribution, logarithmic scale.

Figure 2.5: Calculated cross-section, logarithmic scale.

Conclusion

The general purpose of the work was to validate NuPropagator package and to determine features that are needed to be under development. For the goal Bjorken variables distributions produced by this package were obtained.

During the validation of NuPropagator, LHAPDF and Vegas packages were taken into account, also skills of using such Python packages as Numpy and Scipy were enhanced. Bjorken variables and Python packages which produce them have been described in this paper. Distributions of these variables were also plotted and analysed during my work. During the program various articles on deep inelastic scattering, Bjorken scaling and neutrino interaction simulation were studied.

The further work intentions are to explore different LHAPDF datasets to find the most compatible one. Additionally, I plan to focus on searching the most suitable cross-section integration method.

References

1. Simulation of Ultra High Energy Neutrino Interactions in Ice and Water, S. Bevan et al. for ACoRNE Collaboration, 31 October 2018

2. Neutrino Cross section, Noemi Rocco for 12th International Neutrino Summer School 2019, Fermilab, 13 August 2019

3. QUARKS & LEPTONS, FrancisHalzen and AlanD.Martin, 1984

4. Electroweak interactions : an introduction to the physics of quarks and leptons, Peter Renton, 1990

5. Electron Scattering and Neutrino Physics, A. M. Ankowski et al. A NF06 Contributed White Paper Submitted to the Proceedings of the US Community Study on the Future of Particle Physics (Snowmass 2021)

Acknowledgments