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Dynamics of nuclear capture and fusion in reactions with heavy ions

Supervisor:
Dr. Avazbek Nasirov, BLTP.

Student :
Boyazidkhon Sultonov, Russia, MIPT.

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1. Abstract

Studying the dynamics of capture and fusion in reactions involving heavy ions is a significant area in nuclear physics. This work examines key aspects of this dynamics and current research directions. In particular, the interaction between two heavy nuclei subjected to the forces of Coulomb repulsion and nuclear attraction is described. The potential energy of this system, including both Coulomb and nuclear components, is discussed, along with methods for its modeling. Methods for describing the Coulomb potential for spherical and deformed nuclei are discussed, as well as the application of the proximity potential approximation to describe interactions between nuclear surfaces. The final part of the work is dedicated to the results of numerical simulations, allowing the visualization of nuclear reaction trajectories depending on initial conditions and energetic parameters of the system.

2. Introduction

Studying the dynamics of nuclear capture and fusion in reactions involving heavy ions constitutes a significant area within nuclear physics. These reactions, occurring when heavy nuclei collide, hold not only great scientific interest but also have a wide range of practical applications, including energy production, medicine, and materials science. In this introduction, we will explore the key aspects of the dynamics of nuclear capture and fusion in reactions with heavy ions, as well as examine current research directions in this field.

Reactions involving heavy ions involve the collision of two heavy nuclei, each containing a large number of protons and neutrons. As these nuclei approach each other, various physical phenomena arise, including capture, fusion, quasifission, and emission of different particles. Understanding these processes requires studying the dynamics of nuclear particle interactions and the peculiarities of their potential energy.

The dinuclear system (DNS) formed in heavy-ion collisions is characterized by the interaction between two nuclei, which are subjected to the forces of Coulomb repulsion and nuclear attraction. These forces determine the dynamics of nuclear movement and influence the likelihood of different reaction pathways, including capture and fusion.

One of the main components of the potential energy of the two-nucleus system is Coulomb energy, which depends on the charges of the nuclei and the distance between them. For spherical nuclei, it can be described by Coulomb's law, which governs the interaction of charges in the system. However, when considering nuclei with quadrupole deformation, additional contributions to Coulomb energy associated with the charge moment need to be taken into account.

Additionally, nuclear potential energy plays a crucial role in the dynamics of nuclear reactions, characterizing the interaction between nucleons inside the nucleus. This energy is determined by complex models, such as the definition of the Bass potential and the proximity potential, which account for the features of nucleon structure and interactions in nuclei.

The graphics of these potentials provide a visual representation of the change in potential energy as a function of the distance between nuclei. The sum of Coulomb and nuclear potentials defines the effective potential in which nuclei move during the collision.

Thus, studying the dynamics of nuclear capture and fusion in reactions with heavy ions represents a complex and fascinating area of scientific research. Understanding the physical mechanisms underlying these processes is essential for the advancement of modern nuclear physics and may lead to the development of new promising technologies and applications.

3. Dinuclear system

The nuclear structure, as the foundation of all atomic matter, deeply influences the behavior and interaction of nuclei in a binary nuclear system. Nuclei themselves are complex formations consisting of protons and neutrons, forming nuclear shells. This unique representation has several characteristics that determine their behavior in a binary nuclear system.

One of the main characteristics of nuclei is their atomic number Z and mass number A, respectively. These parameters determine the type of element and its stability. The shape and structure of nuclei also play a significant role in production of the reaction products. Unlike the notion of spherical nuclei, many of them have deformed shapes. Orientation angles of their axial symmetry axis are oriented chaotically in space during collision. The results of the experiment depends on the orientation angles. Their values affect on the the interaction of nuclei of the DNS, especially the Coulomb and nuclear interaction potentials.

The excitation energy of the DNS and its nuclei also play an important role. Nuclei can be in states with energy higher than the ground state, which affects their ability to interact with other nuclei and nucleon transfer between them. They can be caused by the dissipation of the kinetic energy of collisions or the emission of neutrons or protons and gamma-rays. These features of nuclear structure significantly influence the dynamics of the DNS evolution.

The structural characteristics of each nucleus play a determining role in the outcome of the reaction. This can significantly influence the interaction between them, determining whether capture and fusion will occur. Additionally, the shape and deformation of the nucleus also influence on the lifetime of DNS and probability its transformation into compound nucleus. The presence of excited states can also play a significant role in non-equilibrium distribution of the dissipated part of the kinetic energy between reaction products. Thus, the structural characteristics of each nucleus significantly influence the overall behavior of the DNS and determine its reaction pathways.

3.1 Coulomb potential energy

In the world of nuclear physics, one of the fundamental concepts is the interaction between charged particles that make up atomic nuclei. This interaction is determined by the Coulomb potential, which plays a crucial role in the structure and properties of the nucleus. In this work, we will examine the role of the Coulomb potential in nuclear physics and its dependence on nuclear shape.

Let's start by considering the Coulomb potential for nuclei with a spherical shape. In this case, when nuclei have a symmetric shape, the Coulomb potential can be expressed by a simple formula based on the distance between charges and their magnitudes. This potential typically has a form inversely proportional to the distance between the centers of nuclei:

$$V_C = \frac{Z_1 Z_2 e^2}{R} \tag{1}$$

Here, Z_1 and Z_2 are the electric charge numbers of the two nuclei, e is the elementary charge and R is the distance between the mass centers of nuclei.

For nuclei with deformed shapes, such as ellipsoids or other nonspherical forms, the formula for the Coulomb potential becomes more complex due to the collisions with the different orientation angles α_1 and α_2 relative to the beam direction. For deformed nuclei, the potential can be expressed by the following formula:

$$V_C = \frac{Z_1 Z_2 e^2}{R} + \frac{Z_1 Z_2 e^2}{R^3} \cdot f(\theta),$$
$$f(\theta) = \left[\left(\frac{9}{20\pi} \right)^{1/2} \sum_{i=1}^{2} R_{0i}^2 \beta_2^i P_2(\cos\alpha_i) + \frac{3}{7\pi} \sum_{i=1}^{2} R_{0i}^2 \Big(\beta_2^i P_2(\cos\alpha_i) \Big)^2 \right] \quad (2)$$

which is a function that takes into account dependence on the orinetation angles α_1 and α_2. The details of the method are presented in Ref. [1].

Some nuclei may have unusual shapes, such as nuclei with elongated shapes or nuclei with a cluster structure. For such nuclei, the Coulomb potential may have a complex dependence on the geometry and distribution of charges. The formula for non-spherical nuclei can be more intricate and depends on the specific geometry of the nucleus.

The Coulomb potential plays a key role in describing the interaction of charged particles in atomic nuclei. Understanding its dependence on the shape of the nucleus allows for a better understanding of the structure and properties of the nucleus. Further research on the Coulomb potential for various nuclear shapes may lead to new discoveries in nuclear physics.

3.2 Proximity potential

The proximity potential is a widely recognized approach, known for its simplicity and numerous applications in the study of various physical phenomena. It is based on the proximity force theorem, which states that two approaching surfaces interact with each other through a force dependent on the distance between them within the range of 2 to 3 fm [2]. It is a straightforward yet effective method for describing the interaction between surfaces.

Consider two approaching nuclei, between which the proximity potential acts. This potential is a product of two functions: one depends on the shape of the two interacting nuclei or the geometry of the nuclear system, while the other is a universal function $\Phi(\frac{s}{b})$, depending solely on the distance between the half-density surfaces of the fragments. This approach allows for the consideration of the shape and geometry of nuclei when describing their interaction.

To calculate the nuclear part of the total interaction potential, we employ the generalized proximity potential known as Prox77. According to its original version, the interaction potential $V_n^{Prox77}(R)$ [2, 3] between two surfaces is expressed as follows:

$$V_n^{Prox77}(r) = 4\pi\gamma b\bar{R}\Phi\left(\frac{s}{b}\right) \quad MeV \tag{3}$$

where b is the surface width parameter (i.e. $b = a\pi/\sqrt{3}$ with $a = 0.55$ fm) and it has been taken close to 1 fm. $\bar{R}$ denotes the mean curvature radius and has the form

$$\bar{R} = \frac{C_1 C_2}{C_1 + C_2}. \tag{4}$$

Here C_i are the Sussmann central radius of the target and projectile, and it is related to the effective sharp radius R_i as,

$$C_i = R_i\left[1 - \left(\frac{b}{R_i}\right)^2 + \dots\right] \quad i = 1, 2. \tag{5}$$

where R_i is given by semi-empirical formula as s function of the mass number A_i:

$$R_i = 1.2A_i^{1/3} - 0.76 + 0.8A_i^{-1/3} \quad \text{fm} \quad i = 1, 2. \tag{6}$$

In equation (3), the separation distance between the half-density surface of two colliding nuclei s is

$$s = r - C_1 - C_2. \tag{7}$$

The surface energy coefficient γ is defined as a function of neutron/proton excess as follows

$$\gamma - \gamma_0[1 - k_s A_s] \quad \text{MeV/fm}^2 \tag{8}$$

where A_s is the asymmetry parameter for the compound nucleus, which means drastic reduction in the magnitude of the potential with asymmetry of the colliding pair. It can be defined as

$$A_s = \frac{(N_1 + N_2) - (Z_1 + Z_2)}{A_1 + A_2} \tag{9}$$

where Z_i and N_i are the proton and neutron numbers of target/projectile nuclei, respectively. $\gamma_0 = 0.9517$ MeV fm$^{-}2$ and $k_s = 1.7826$. Universal function of equation proximity potential given by:

$$\Phi(\epsilon) = \begin{cases} -\frac{1}{2}(\epsilon - 2.54)^2 - 0.085(\epsilon - 2.54)^3, & \text{for } \epsilon \leq 1.2511. \\ -3.437\exp(-\epsilon/0.75), & \text{for } \epsilon \geq 1.2511. \end{cases} \tag{10}$$

The proximity potential is an effective tool for describing the interaction between surfaces in various physical phenomena. Its simplicity and universality make it a widely used method in scientific research.

4. Results

We have developed a Python code to simulate the interaction between nuclei and analyze their trajectories. In this section, we will discuss the results of the simulation presented in the plots.

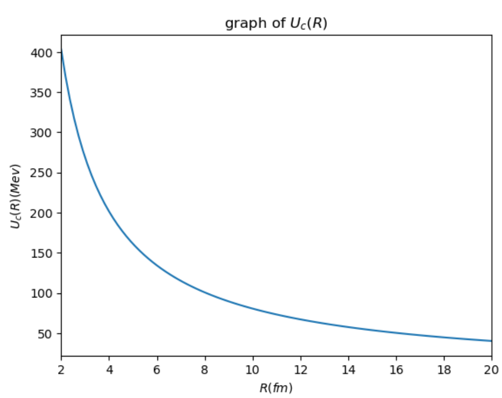

Fig 1. Coulomb potential calculated for the ^{48}Ca+^{72}Ni reaction.

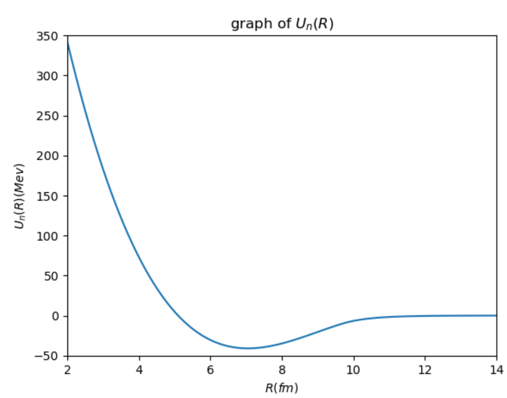

Fig 2. Proximity potential calculated for the ^{48}Ca+^{72}Ni reaction.

First, we plotted the Coulomb and proximity potential energies for two interacting nuclei with charges $Z_1 = 20$ and $Z_2 = 28$ respectively, and atomic masses $A_1 = 48$ and $A_2 = 72$ in Figs. 1 and 2, respectively. The graphs display the values of these potentials as a function of the distance between the nuclei.

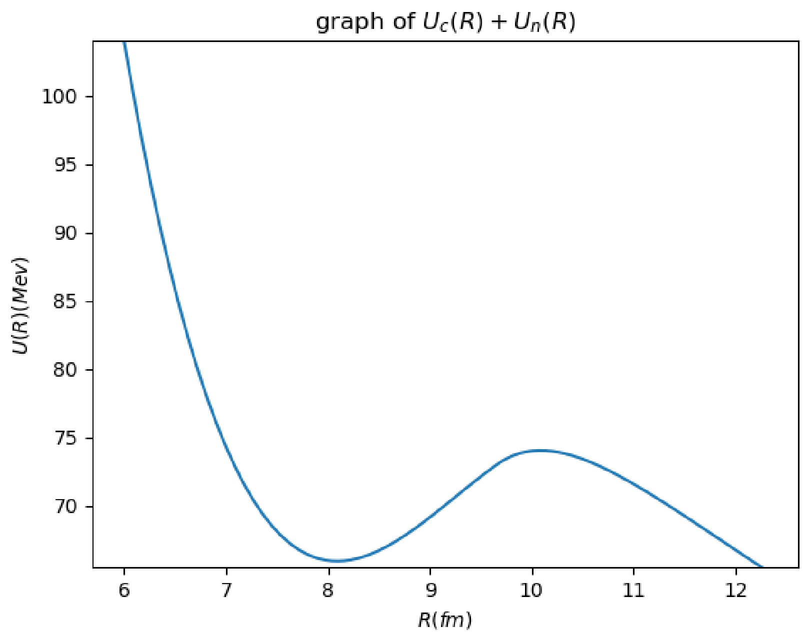

Fig 3. Total interaction potential $U(R)$ calculated for the ^{48}Ca+^{72}Ni reaction.

Fig. 3 shows the sum of two potentials $U(R) = U_C(R) + U_n$ presented in Figs. 1 and 2.

As observed from the graphs, both Coulomb and proximity potential energies significantly influence the dynamics of nuclear interactions. The proximity potential energy allows for the consideration of nuclear shape and geometry, making it more versatile and effective for describing interactions at small distances.

By solving the equations of motion,

$$\mu\ddot{R}(t) + \gamma_{R(t)}\dot{R}(t) = -\frac{dU(R(t))}{dR(t)}, \tag{11}$$

here μ- reduced mass of the DNS and $\gamma_{R(t)}$- function of friction, we can model and display the trajectories of the colliding nuclei at different collision energies $E_{c.m.}$. These trajectories reflect changes in the positions of the nuclei in space over time and allow us to visualize their behavior under various energetic conditions.

Using numerical methods to solve the differential equations of motion, we can analyze the influence of various factors such as Coulomb interaction, proximity potential, frictional force, and others on the motion of nuclei. By adjusting parameters such as $E_{c.m.}$ and initial conditions, we can explore different scenarios and predict their outcomes. Next, we present graphs of the solutions to

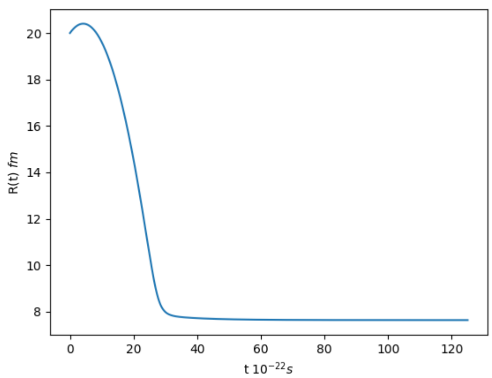

Fig 4. Solution of the equation of motion for the dependence of the distance between the colliding centers nuclei versus time for collision energy $E_{c.m.} = 250$ MeV.

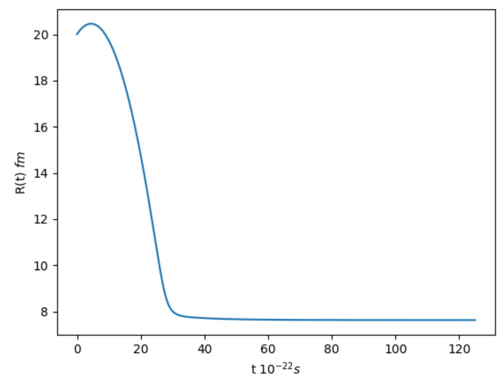

Fig 5. Solution of the equation of motion for the dependence of the distance between the colliding centers nuclei versus time for collision energy $E_{c.m.} = 275$ MeV.

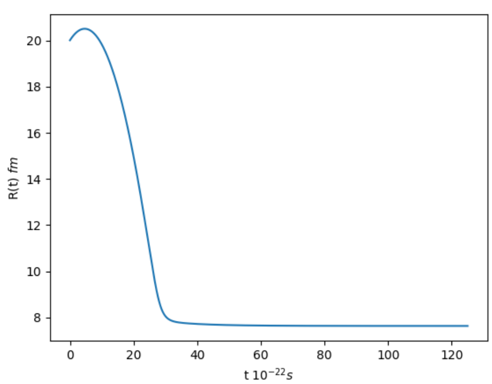

Fig 6. Solution of the equation of motion for the dependence of the distance between the colliding centers nuclei versus time for collision energy $E_{c.m.} = 300$ MeV.

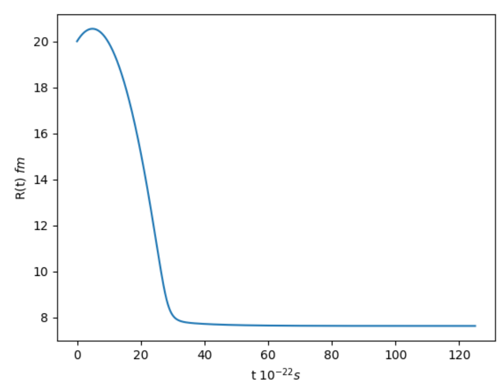

Fig 7. Solution of the equation of motion for the dependence of the distance between the colliding centers nuclei versus time for collision energy $E_{c.m.} = 325$ MeV.

the equations of motion for four different energies of the system's center of mass. This allows us to visually observe how the trajectory of nuclear motion changes depending on the initial conditions and energetic parameters of the system. The boundary condition for the radial distance is given as $R(t = 0)$=20 fm and initial energy of the projectile nucleus (at $R \to \infty$) was given as $E_{c.m.}$=250, 275, 300 325 MeV. The results are presented in Figs. 4, 5, 6 and 7, respectively.

It is seen that capture of projectile by target nucleus occurs at the considered energies of collision.

Upon closer examination of the graphs, it can be observed that the higher the energy $E_{c.m.}$, the further the graph shifts to the right. The nuclei approach each other until the distance between them becomes approximately equal to the sum of their radii. On your graphs, capture occurs because the nuclei are approaching each other, and the kinetic energy decreases due to frictional forces, becoming less than the height of the potential well. Therefore, further change in the distance between them is not observed over time.

5. Conclusion

Studying the dynamics of nuclear capture and fusion in reactions involving heavy ions is an important area in nuclear physics with both scientific and practical significance. The results of our numerical simulations provide valuable insights into the trajectories of nuclear interactions depending on initial conditions and energy. Visualizing these trajectories allows us to observe how the dynamics of nuclear motion interacts with factors such as Coulomb interaction, proximity potential, and friction. Furthermore, we have demonstrated the importance of considering structural characteristics of nuclei, such as charge distribution and deformation, in predicting reaction outcomes. We anticipate that considering the potential for rotation may reveal even more remarkable physical phenomena, highlighting the importance of further research in this field.

6. Acknowledgements

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