JOINT INSTITUTE FOR NUCLEAR RESEARCH
Veksler and Baldin laboratory of High Energy Physics

FINAL REPORT ON THE STUDENT PROGRAM

Correlation functions for pions

Supervisor: Alexey A. Aparin
Student: Pishchaeva V. Anna
Russia, Ural Federal University

Participation period: 12 March - 29 April

Dubna, 2023

Abstract

The main goal of this report is to study quark gluon plasma (QGP) which, in our case, is produced in Au+Au collisions at energies of $\sqrt{s_{NN}} = 200$ GeV. In such conditions, correlations of physical characteristics (momentum and energies) of particles occur. Therefore, by looking at those correlations, using correlation functions, we have an opportunity to explore properties of QGP. We will study correlation functions for pions, using data from the STAR detector. In order to accomplish this task, the identification of the desired kind of hadrons is needed as well as the analysis of the track effects on the form of the correlation function.

Contents

1 Introduction

1.1 Quark Gluon Plasma

The phase of deconfined quarks and gluons, a.k.a. quark gluon plasma (QGP), appears at high temperature and/or at high matter density, which can be achieved using the collision of heavy nuclei at high energies. Thus, in such conditions, quarks and gluons become asymptotically free from strong nuclear force constraining their movement within a nucleon. [1]

After the collision, QGP undergoes multiple stages. The initial stage is a pre-equilibrium one where partons are freed because of each nucleon's scattering. Those partons (quarks and gluons) re-scatter, reaching equilibrium in the process and creating thermalized quark gluon plasma. The plasma expands collectively, cooling down, which leads to the formation of hadrons from quarks and gluons (hadronization) [2]. Consequently, QGP reaches the point of chemical freeze-out, where the total number of particles and their kinds do not change. Finally, when hadrons' momentum cease changing, the kinetic freeze-out is reached.[4]

In this work, we use the experimental data which were produced in Au+Au collisions at energies of $\sqrt{s_{NN}} = 200$ GeV in the STAR experiment. Particularly, we are interested in working with the data of pions.

1.2 STAR experiment

For our analysis, we are going to consider information from two parts of the STAR detector: TPC and TOF.

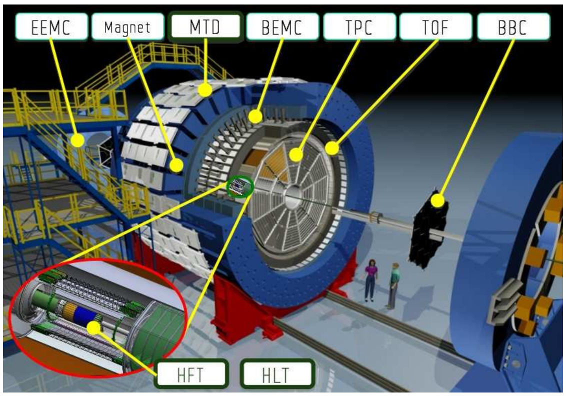


Figure 1: the STAR detector

The TPC is the time projection chamber, which is filled with 10% methane and 90% argon. It measures the space points of a particle's path as well as the ionization of the gas. The combination of a particle's trajectory and the value of the magnetic field give an opportunity to evaluate the momentum and rigidity of a particle, which, in turn, leads to the discovery of a particle's charge.

TOF is the time-of-flight detector that allows us to find the velocity of a particle. With this knowledge, it is possible to identify a particle's mass.

Thus, using data from these detectors, we will be able to extract pions from the pool of other hadrons, which will allow us to build correlation functions for this kind of particles.

1.3 Correlation function

In the process of QGP expansion, pairs of hadrons occur. At different moments of the expansion, there is a different number of hadron pairs of given momentum. Therefore, a function that would consider the probability of pair occurrence could help identify the region of homogeneity — the source producing particle pairs within a portion of momentum phase space. [3]

That function is a correlation function which is described by the next formula:

$$C(\vec{q},\vec{k}) = \frac{P(\vec{p_1},\vec{p_2})}{P(\vec{p_1})P(\vec{p_2})}, \tag{1.1.1}$$

where $P(\vec{p_1},\vec{p_2})$ - the probability of detecting two particles with momentum $\vec{p_1}$ and $\vec{p_2}$; $P(\vec{p_1})$ - the total probability of a source emitting a particle with momentum $\vec{p_1}$; $P(\vec{p_2})$ - is the total probability of a source emitting a particle with momentum $\vec{p_2}$.[1]

Knowing what kind of interaction occurs between particle pairs and knowing the correlation function, we can identify the size of the source.

However, in order to find the experimental value of the correlation function, we will need to use another formula, the experimental one:

$$C(\vec{q},\vec{k}) = \frac{N(\vec{q})}{D(\vec{q})}, \tag{1.1.2}$$

where $N(\vec{q})$ - a numerator; $D(\vec{q})$ - a denominator.

This formula allows us to separate the signal from the background. The idea behind this method is to use tracks from different events for the denominator. Thus, such mixed tracks don't have any real interaction - the background. As for the numerator, we consider tracks from the same event to capture the signal.

The experimental data for such an evaluation was collected in the STAR experiment.

2 Analysis of the data from the STAR detector

The data that we acquire from the STAR detector consists of physical quantities that describe either a collision of heavy nuclei itself or the charged particles which were produced in this collision. The former is called an "event", which includes such quantities as the position of a collision of two nuclei (the primary vertex position), reference multiplicity, and etc. The properties of occurred hadrons are described by "tracks". They include particle's momentum, number of hits of its track in TPC, the minimum distance between its track and the primary vertex position of the event (DCA), and etc.

In order to work with pions, we first need to differentiate them from other particles by making some cuts to different events' and tracks' characteristics.

2.1 Event selection

First, we will select events, which were detected with the maximum precision, by applying cuts on them.

The cuts on events include the following:

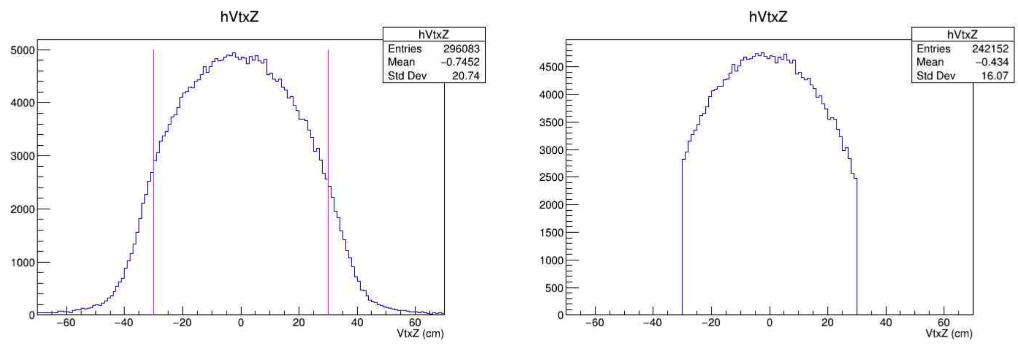
- $|V_z| < 30\ (cm)\ cut$ - the cut on the primary vertex position of an event in the z-direction
- $|V_r| < 2\ (cm)\ cut$ - the cut on the primary vertex position of an event in the radial direction

- $|V_{zdiff}| < 3\ (cm)\ cut$, where $|V_{zdiff}| = |V_{zTPC} - V_{zVPD}|$, the cut on the difference of the primary vertex position of an event in the z-direction obtained from TPC and from the VPD. The VPD is the vertex position detector which exists as two identical assemblies located on each side of the STAR detector very close to the beam pipe. VPD allows us to assess the primary vertex position of an event.

The reason for each cut is explained below.

a) The cut on the primary vertex position of an event in the z-direction ($|V_z| < 30\ (cm)$)

This cut is done on the position of the primary vertex. In the Cartesian coordinate system, there are x, y, and z coordinates of the vertex. The z-coordinate is a longitudinal component that is responsible for the position of the primary vertex along the beam pipe of the STAR detector. On the edge of the detector, after nuclei collision, the portion of formed hadrons flies out of the detector, making the multiplicity of an event lesser than an actual event's multiplicity, which may lead to incorrect identification of the event's centrality. Therefore, to ensure that the detected number of formed hadrons is as close to the real number of particles as possible, we are doing the cut on Vz. The below images demonstrate the distribution of Vz before any cuts and after the cuts:

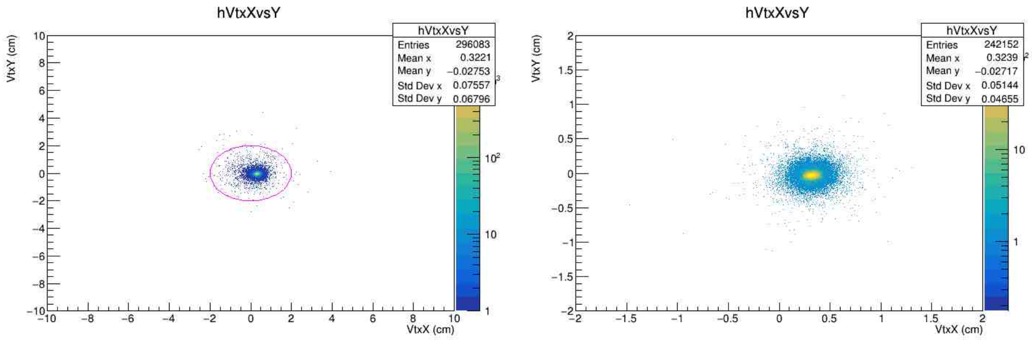


a) b)

Figure 2: The primary vertex of z-coordinate. a) V_z without cuts b) V_z with events' cuts

b) The cut on the primary vertex position of an event in the radial direction ($|V_r| < 2\ (cm)$)

The loss of formed particles on the edges of the detector is not the only problem that the position of the primary vertex introduces. In the transverse direction, if the primary vertex gets closer to the beam pipe, the collision of nuclei may be disrupted by one of the nuclei interacting with the beam pipe's material. Therefore, the formed particles are not going to be from the Au+Au collision, which is not what we are considering. Therefore, we need to get rid of such a predicament by applying the above cut:



a) b)

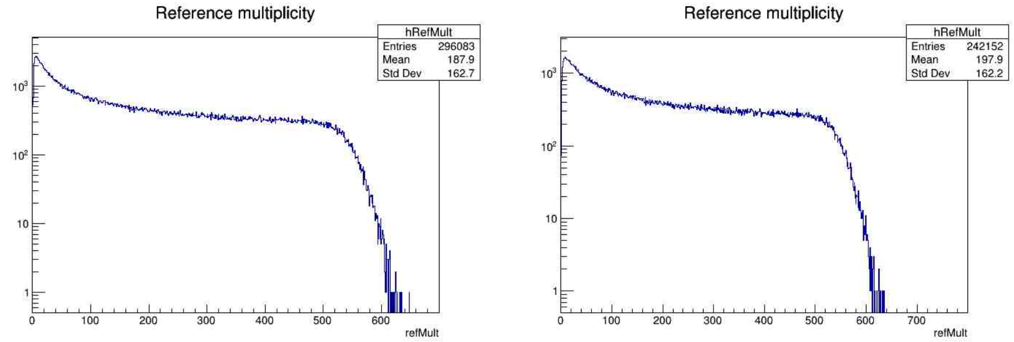
Figure 3: Primary vertex in xy-plane. a) V_r without cuts b) V_r with events' cuts

c) The cut on the difference of the primary vertex position of an event in the z-direction obtained from the TPC and from the VPD ($|V_{zdiff}| < 3\ (cm)$).

$$|V_{zdiff}| = |V_{zTPC} - V_{zVPD}|,$$

where V_{zVPD} is defined in the VPD; V_{zTPC} is defined in the TPC. Therefore, we have two values of the primary vertex from different detectors. This condition allows us to select events with the better defined primary vertex.

The cuts on the events influence reference multiplicity. Reference multiplicity is connected to the event's centrality which describes the initial overlap region of the colliding nuclei. In the further analysis, we will divide our correlation functions based on centrality, assuming that the geometry of the initial overlap effects the homogeneity region.



a) b)

Figure 4: Reference multiplicity. a) RefMult without cuts b) RefMult with events' cuts

The number of entries in reference multiplicity is less than the number before the cuts, since some of the events and their formed hadrons were cut off.

2.2 Track selection

After we aqcuired the "good" events, we will need to obtain the "good" tracks of produced charged particles in these events. Using tracks' cuts, we will not only get more accurate data for our analysis but also identify the right kind of particles for the construction of correlation functions.

Thus, there are cuts on tracks that will allow us to get accurate representation of charged particle's properties:

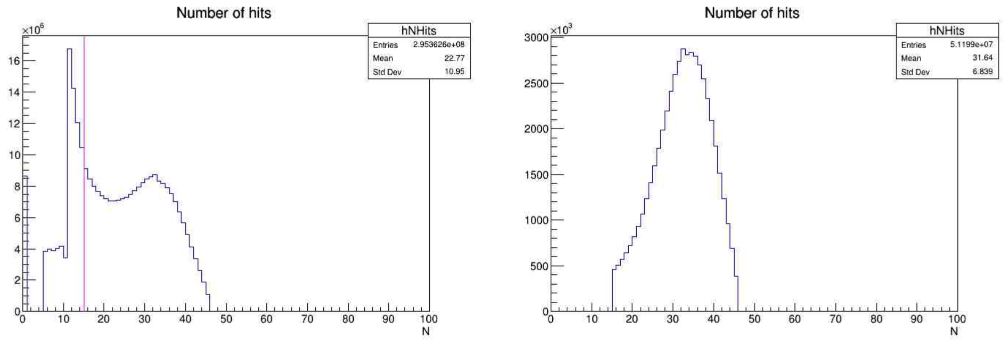
- $NHits < 15$ - cut on track's hit in TPC
- $|\eta| < 1$ - cut on pseudorapidity
- $0.15 < p_{tot} < 1.45$ (GeV/c) - cut on the primary total momentum of a particle
- $0.15 < p_T < 1.45$ (GeV/c) - cut on the primary transverse momentum of a particle
- $DCA < 3$ (cm) - cut on the least distance between the primary vertex of an event and a particle's track

The reason for such cuts is described as following:

a) The cut on the number of track's hits in the TPC ($NHits < 15$)

This cut allows us to construct the track of a particle with the sufficient accuracy. The accuracy of the trajectory matters since the constructed track is used to identify the particle's momentum.

The following figure demonstrate the change in the plot of the number of hits before and after the cuts:



a) b)

Figure 5: Number of hits produced by a particle's track. a) NHits without cuts b) NHits with tracks' cuts

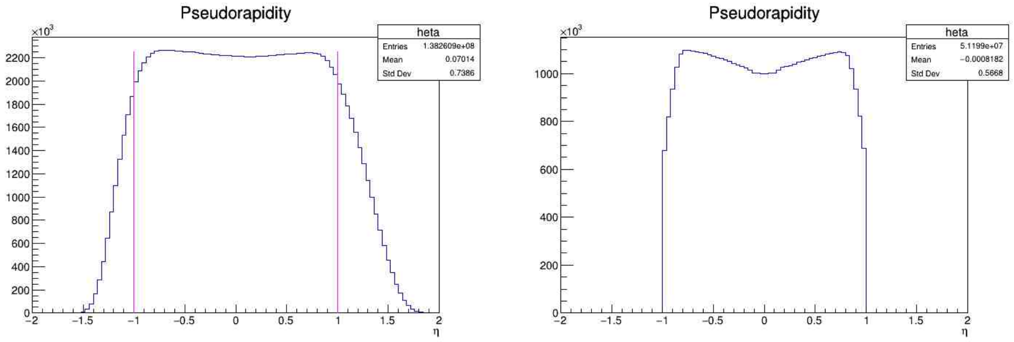
b) The cut on pseudorapidity ($|\eta| < 1$)

Pseudorapidity is found, using the following formula:

$$\eta \equiv -\ln\left[\tan(\frac{\theta}{2})\right],$$

where θ is the angle of a particle relative to the beam axis.

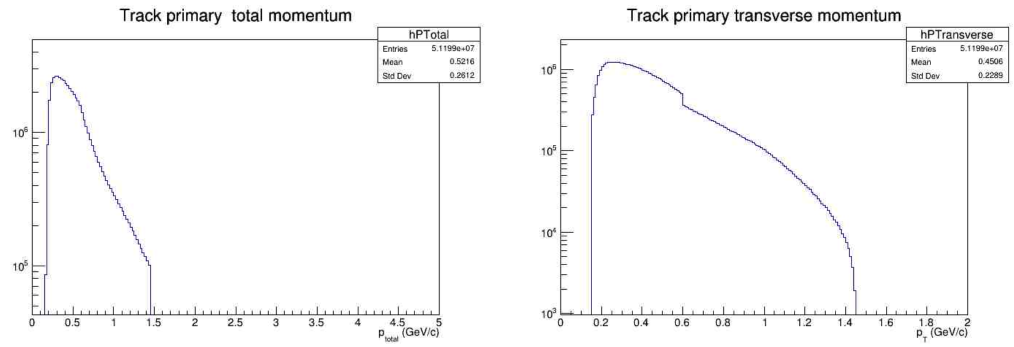
This cut allows us to select tracks that are almost perpendicular to the beam line. Therefore, such tracks are detected by the TPC and TOF more effectively.



a) b)

Figure 6: Pseudorapidity of a particle. a) η without cuts b) η with tracks' cuts

c) The cut on primary momentum ($0.15 < p_{tot} < 1.45$ (GeV/c) and $0.15 < p_T < 1.45$ (GeV/c)).

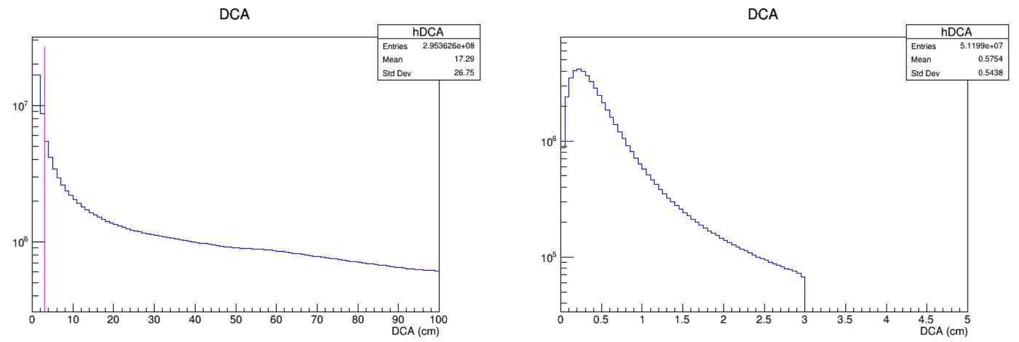


a) b)

Figure 7: Momentum of a particle after the cuts. a) p_{tot} b) p_T

d) The cut on the least distance between the primary vertex of an event and a particle's track $DCA < 3$ (cm)

This condition enables us to take into account only primary tracks, i.e. tracks of particles that occurred from the Au+Au collision and not from the products of the nuclei collision:



a) b)

Figure 8: DCA of a particle. a) DCA without cuts b) DCA with tracks' cuts

There are also cuts which are considered for particle's identification. Identification of hadrons is based on the loss of particle's energy in the TPC due to the ionization of the gas. The loss is described by Gaussian distribution, where we have a parameter σ defining the deviation from this distribution.

In order to identify $\pi\pi$, we will use the particular conditions from the TPC and TOF - the PID (particle identification). The conditions in the PID are the following:

1) TPC: $if\ p_{tot} < 0.6\ (GeV/c)\ |nSigmaPart| < 2,\ |nSigmaOther| > 2$ - where nSigmaPart is σ of pions, and nSigmaOther is σ of other kinds of hadrons.

2) TPC+TOF:

   - $else\ if\ \beta > 0\ |invBetaDiff| < 0.015,\ |nSigmaPart| < 3$ - where $|invBetaDiff|$ is:

$$|invBetaDiff| = \left|\frac{1}{\beta_{expected}} - \frac{1}{\beta_{TOF}}\right|,$$

   where β_{TOF} is β obtained from TOF; $\beta_{expected}$ is found, using the next formula:

$$\beta_{expected} = \sqrt{\frac{p_{tot}^2}{p_{tot}^2 + m^2}},$$

   where p_{tot}^2 is the square of the primary total momentum of a particle; m^2 is the square of the particle's mass.

   - AND $if\ -0.02 < m^2 < 0.08\ (GeV)^2$

We use the TPC when $p_{tot} < 0.6$ (GeV/c) without TOF since TOF does not detect particles with low primary momentum effectively.

The reason for such PID cuts is following:

1) TPC without TOF

- $if\ p_{tot} < 0.6$ (GeV/c). This cut ensures that we will be able to differentiate each particle, since after the primary total momentum of hadrons become more than 0.6 GeV/c, different kinds of particles get mixed up.
- $|nSigmaPart| < 2$ restricts us to consideration of pions since their deviation from Gaussian distribution is low.
- $|nSigmaOther| > 2$ helps us not to look at the other kind of particles.

2) TPC with TOF

- $-0.02 < m^2 < 0.08$ selects particles with masses that are close to the mass of pions. Mass is found with the formula:

$$m^2 = p_{tot}^2 * (\frac{1}{\beta^2} - 1)$$

  where β, the velocity of a particle over the speed of light, is known from TOF; p_{tot}^2 is the square of the primary total momentum of a particle $(\mathrm{GeV/c})^2$. The reason why mass can be less than zero is because of the way TOF registers β. For particles that hit the TOF detector in an almost perpendicular manner, the time of the signal is going to be small, which will lead the detector to write down the great value for the velocity. It may appear that this velocity, because of such peculiarities, is greater than the speed of light, introducing the negative mass.

- $\beta > 0$ provides that a particle has TOF.
- $invBetaDiff < 0.015$.

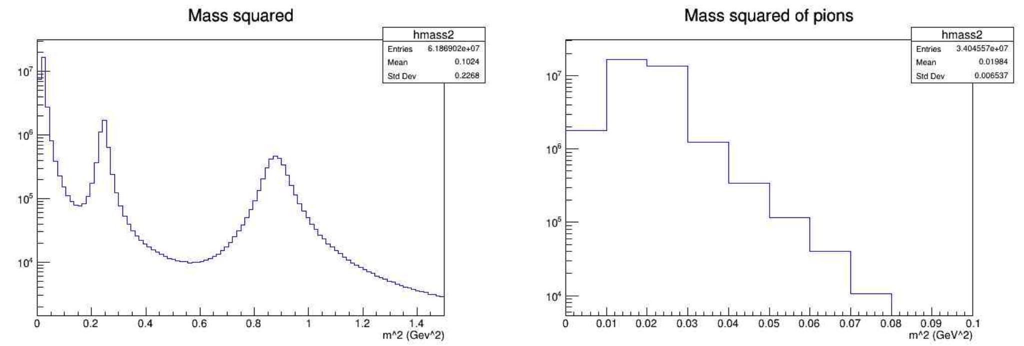
  This condition selects particles whose β close to the expected β for pions.

- $|nSigmaPart| < 3$.

  The previous condition is not enough to differentiate the desired particles from the rest. TOF has a restricted number of detecting panels, which, in turn, poses a problem when multiple primary particles hit the same panel. It may lead to the identification of an incorrect β which may coincide with the expected beta of the desired particle. To ensure that we are definitely observing pions, we apply this condition.

The resulted distribution of miscellaneous physical characteristics of charged particles before and after the above described cuts:

a) Mass squared (m^2):

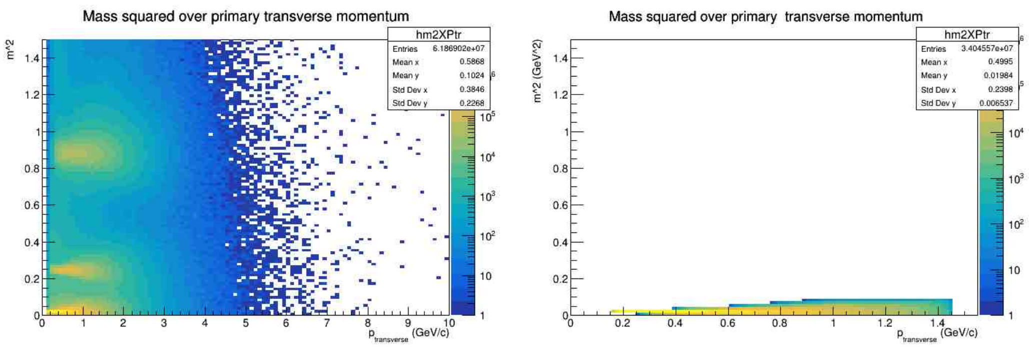


a) b)

Figure 9: m^2 of a particle. a) m^2 without cuts b) m^2 with tracks' cuts

Thus, we are looking at masses which are close to the one of a pion

b) Mass squared over the primary transverse momentum ($m^2(p_T)$):



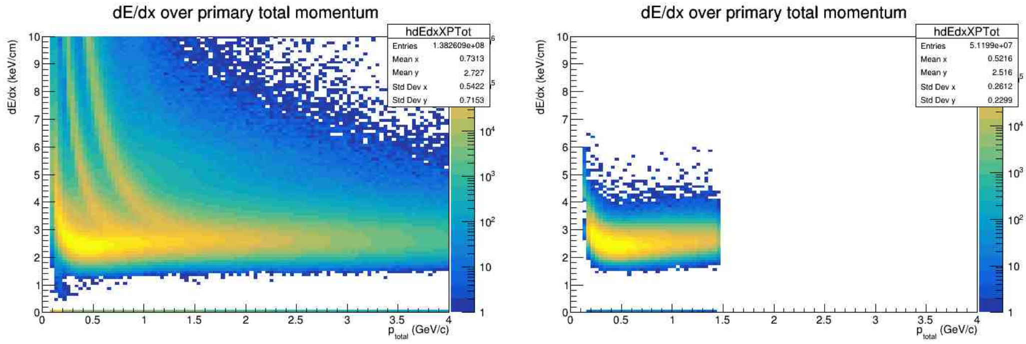
a) b)

Figure 10: m^2 *over* p_T. a) without cuts b) with tracks' cuts

The same logic applies as with the previous figure 9: we can see that, in general, we consider particles that coincide with pions.

c) Ionization over primary total momentum $dE/dx(p_{tot})$.

On the ionization histogram (Figure 11), the closest yellow line to the y-axis mainly consists of pions. Through all the cuts, we were able to identify it and get rid of all the other unnecessary particles for this analysis.

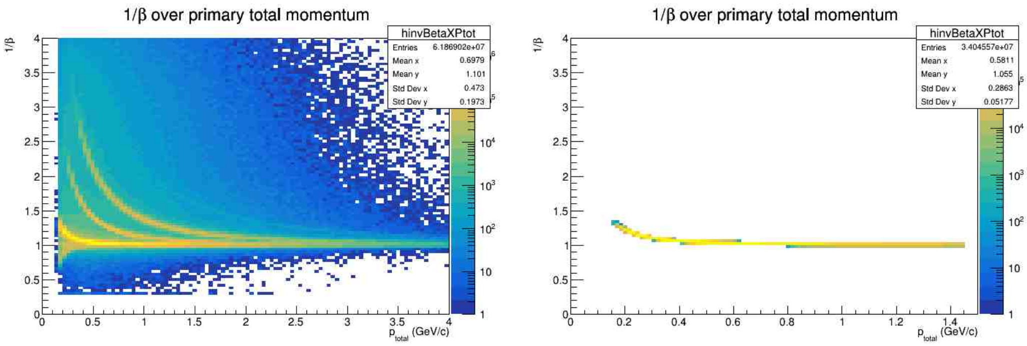


a) b)

Figure 11: Ionization histogram a) without cuts b) with tracks' cuts

d) The dependence of the ratio of speed of light over the particle's velocity on primary total momentum ($1/\beta(p_{tot})$).

It should be noted that for different types of particles, there is a different yellow line in the histogram. With the same momentum, particles with a smaller mass have a bigger velocity, meaning a bigger β, which leads to the smaller $1/\beta$. Therefore, the lightest particles are at the bottom of the graph, which are electrons. Right above them go the desired hadrons— pions.



a) b)

Figure 12: Histogram of $1/\beta$ over p_{tot} with tracks' cuts

3 Correlation functions

When identification was done, and we acquired pions, we can now find the correlation function for $\pi\pi$, using the formula 1.1.2, where the numerator and the denominator appear to be:

$$q_{inv} = \sqrt{(\vec{p}_{1tot} - \vec{p}_{2tot})^2 - (E_1 - E_2)^2}, \tag{3.1}$$

where $\vec{p}_{1tot}$, E_1 are the total primary momentum and the energy of a first particle; $\vec{p}_{2tot}$,E_2 are the total primary momentum and the energy of a second particle.

Even though the numerator and the denominator have the same formula, the primary momentum and energies of pairs in the numerator are taken from the same event, whereas in the denominator they are from separate events.

Correlation functions are built for different charges, centrality, momentum, and, for the denominator, we also take into account the primary vertex position in the z-direction.

The reason why we consider V_z for the denominator is to reduce the effect of the detectors, which may appear if V_z of particles differs noticeably. It is not considered for the numerator, since there is the same event that has the same primary vertex position for all tracks. However, for the denominator, we consider mix events that have different z-positions of the primary vertex which differ from each other on the value not greater than 5 cm, not forgetting that our interval for the z-positions of the primary vertex is (-30, 30) cm.

The other division, which is related to the centrality of an event, is done on both the numerator and the denominator of a correlation function.

Centrality is divided in next intervals:

- $0 - 10\%$
- $10 - 30\%$
- $30 - 80\%$

The last aspect which we will consider for a correlation function is k_T.

k_T is found, using formula:

$$k_T = \left| \frac{\vec{p_{1T}} + \vec{p_{2T}}}{2} \right|,$$

where $\vec{p_{1T}}$, $\vec{p_{2T}}$ - primary transverse momentum of particles in pairs.

As was mentioned before, a correlation function measures the size of a homogeneity region, where particles have close values of momentum. Thus, we consider pairs of the same k_T, which approximates a source size to the size of a homogeneity region.

k_T is divided in next intervals:

- $0.15 < k_T < 0.45$ (GeV/c)
- $0.45 < k_T < 0.75$ (GeV/c)
- $0.75 < k_T < 1.05$ (GeV/c)

3.1 Correlation function with no consideration of track effects

Track effects can alter the form of the correlation functions. For starters, we want to see how they look without acknowledging track effects.

Correlation functions for $\pi^+\pi^+$ and $\pi^-\pi^-$ with fixed centrality $(30-80\%)$ with different k_T:

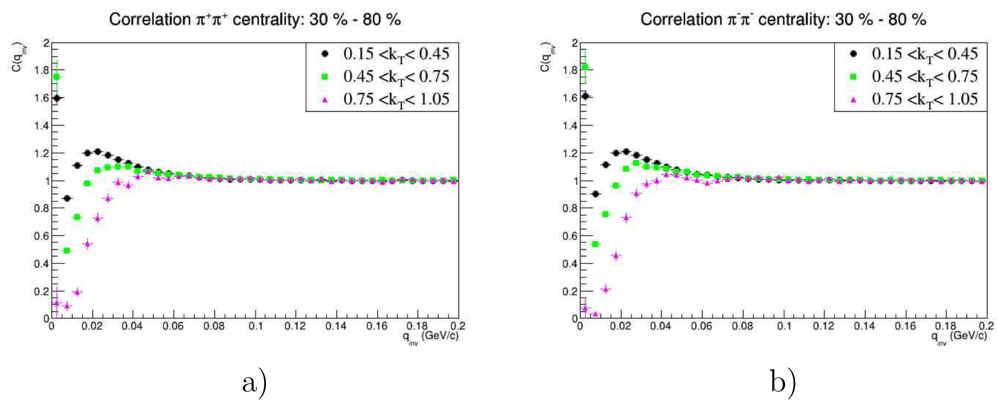
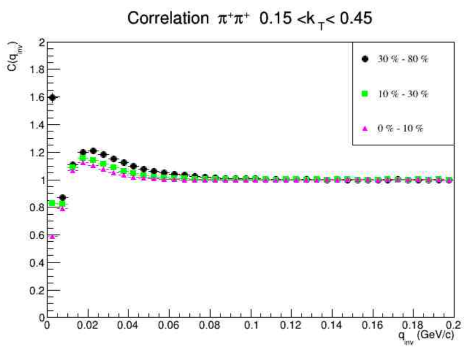


Figure 13: Correlation function with no consideration of track effects for centrality of $(30-80\%)$. a) $\pi^+\pi^+$ b) $\pi^-\pi^-$

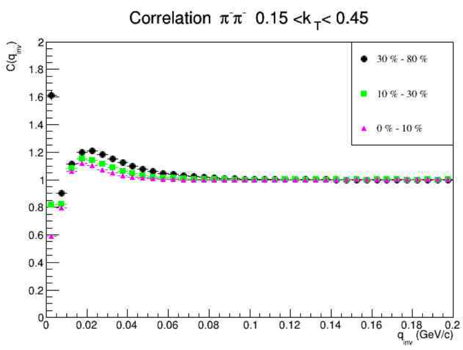
As we can see, the fall of the correlation functions starts differently for different k_T. The fall demonstrates the correlation of the pairs, since the numerator representing the probability of the pair and the denominator representing background commences to differ from each other. For the lowest

k_T $(0.15 < k_T < 0.45)$ GeV/c the fall is the latest one, and for the largest k_T $(0.75 < k_T < 1.05)$ GeV/c the fall is the earliest one. Using the uncertainty principle of Heisenberg, we can understand the reason behind such behavior. When the fall is the early one, the distance to the start of the fall is large - the large value for the momentum. The momentum is reverse proportional to the position (the Heisenberg principle), the large value of the momentum means the low value for the position. Therefore, for the large k_T we have the low source size and vice versa. Meaning, that particle pairs with large k_T is flying out from the source on its early stage. Particles with lower k_T are detected when the source is expanding, increasing its size.

Correlation functions for $\pi^+\pi^+$ and $\pi^-\pi^-$ with fixed k_T $(0.15 < k_T < 0.45)$ GeV/c with different centrality:



a)



b)

Figure 14: Correlation function with no consideration of track effects for k_T of $(0.15 < k_T < 0.45)$ GeV/c. a) $\pi^+\pi^+$ b) $\pi^-\pi^-$

3.2 Correlation function with consideration of track splitting

When one track is presented as two tracks, we encounter track splitting. Therefore, the number of pairs that would have similar momentum and energy increases in small q_{inv} (since energies and momentum of two tracks that are constructed from one track are similar to each other), and the correlation function has a sudden increase in small bins of q_{inv}.

Thus, a correlation function with track splitting does not accurately represent a correlation which occurs between particles in a source, meaning, that we need to take track splitting into account. In order to do that, we are going to apply the Splitting Level.

$$SL \equiv \frac{\sum_i S_i}{Nhits_1 + Nhits_2} \quad where\ S_i =$$

$$\begin{cases} +1 & one\ track\ leaves\ a\ hit\ on\ pad-row \\ -1 & both\ track\ leave\ a\ hit\ on\ pad-row \\ 0 & neither\ track\ leaves\ a\ hit\ on\ pad-row \end{cases}$$

SL is applied to both the numerator and the denominator. However, in the numerator, it has a more meaningful role. Since we look at pairs from the same event, it leads to more probable occurrence of track splitting in the numerator than in the denominator. In the latter, it is obvious that two tracks cannot be from splitting. However, we still apply the method to the denominator for the symmetry and consider what if those tracks were in the same event, would they be considered from track-splitting.

We will look at four divisions of SL:

- $SL < 1.0$
- $SL < 0.8$
- $SL < 0.6$
- $SL < 0.4$

Correlation functions for $\pi^+\pi^+$ and $\pi^-\pi^-$ with fixed k_T ($0.15 < k_T < 0.45$)GeV/c and centrality of ($30-80\%$) with different SL:

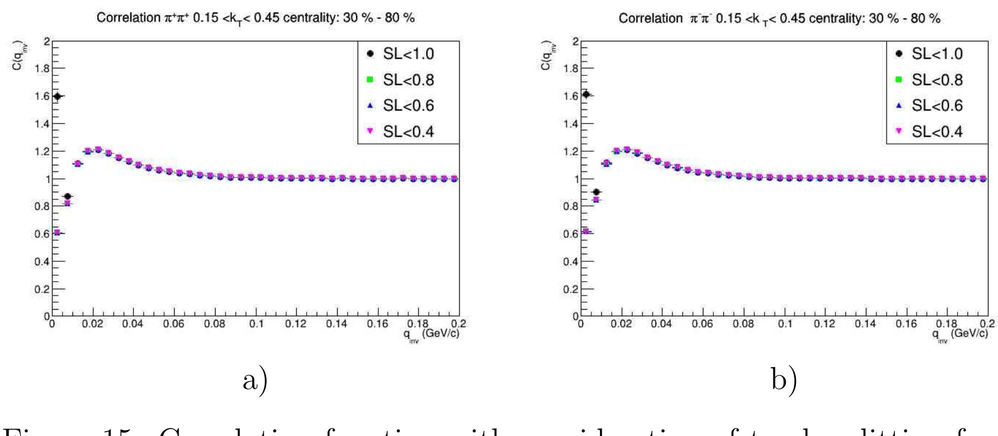
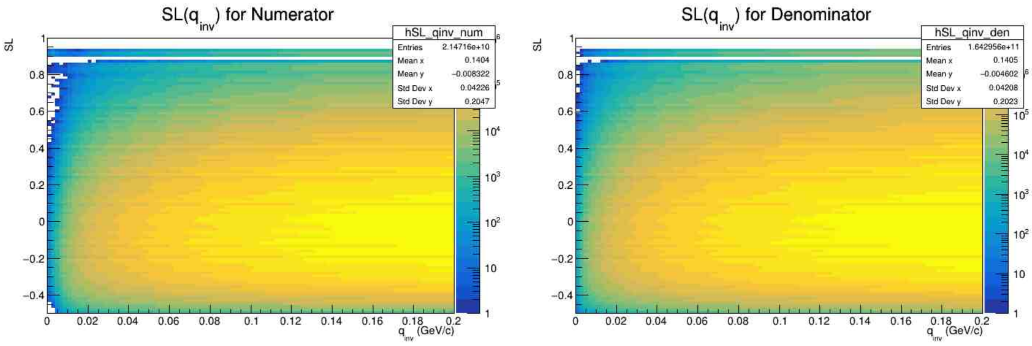


Figure 15: Correlation function with consideration of track splitting for k_T of ($0.15 < k_T < 0.45$) GeV/c and centrality of ($30-80\%$) a) $\pi^+\pi^+$ b) $\pi^-\pi^-$

From figures we can interpret that applying SL decreases the track splitting effect. From figure 16 of SL(q_{inv})



a) b)

Figure 16: SL for a) numerator b) denominator

we see that we can have $SL < 0.6$ without cutting a lot of statistics and making sure that track splitting is accounted for.

3.3 Correlation function with consideration of track merging

When two tracks are represented as one in a detector - it's called track merging. It affects a correlation function since it reduces the number of pairs that have similar momentum and energies. Thus, we need to consider this detector's effect in order to get more accurate depiction of a correlation that occurs in a homogeneity region.

The method of the average distance helps to get rid of track merging.

This method is prevalent in both the numerator and the denominator. However, it has the most importance in the last one. The reason is that when we look at the tracks from mixed events, we may suggest that if those tracks were from the same event, would they be merged? Since we may find many tracks from mixed events that have similar momentum and energy, they would be considered the merged ones.

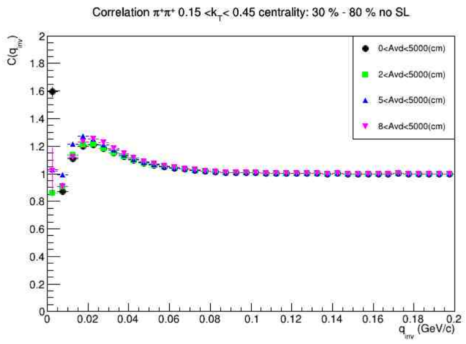
There are several intervals in the Average distance (Avd):

- $Avd > 0$ (cm)

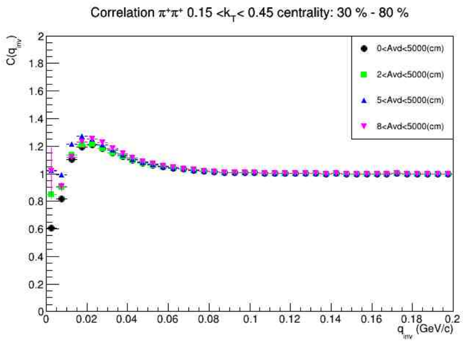
- $Avd > 2$ (cm)
- $Avd > 5$ (cm)
- $Avd > 8$ (cm)

The comparison between correlation functions with SL and without it is also demonstrative of how SL helps with track splitting.

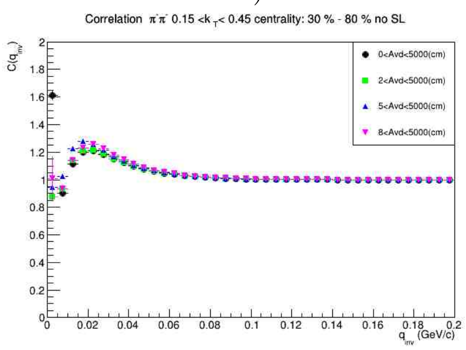
Correlation functions for $\pi^+\pi^+$ and $\pi^-\pi^-$ with fixed k_T $(0.15 < k_T < 0.45)$ GeV/c and centrality of $(30-80\%)$ with different Average distances:



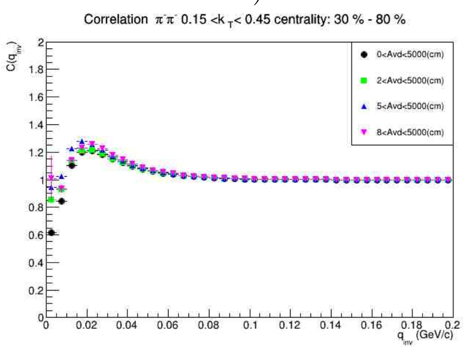
a)



b)



c)



d)

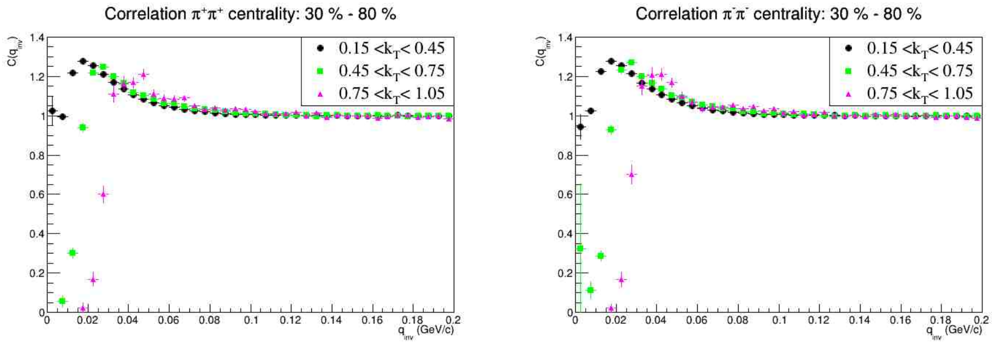
Figure 17: Correlation function with consideration of track splitting for k_T of $(0.15 < k_T < 0.45)$ GeV/c and centrality of $(30-80\%)$ a) $\pi^+\pi^+$ no SL b) $\pi^+\pi^+$ with SL< 0.6 c) $\pi^-\pi^-$ no SL d) $\pi^-\pi^-$ with SL< 0.6

As we can see, with SL, the increase in the first bins of q_{inv} vanishes. It gives a sense of the reduction of the track splitting effect.

We take Avd to be more than 5 cm.

3.4 The final correlation functions

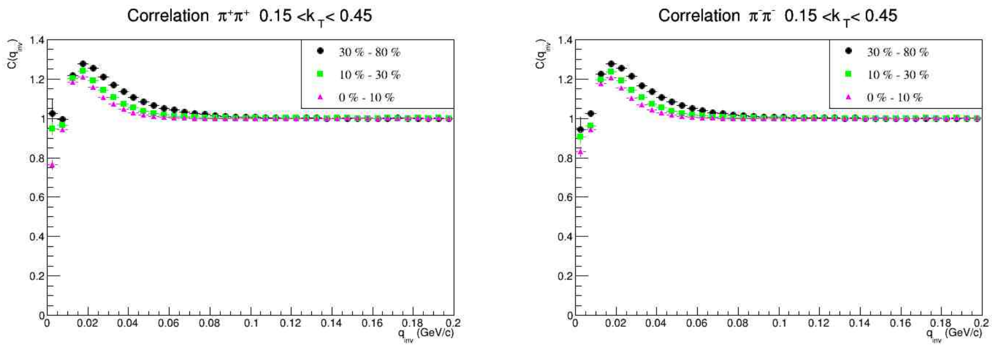
Correlation functions for $\pi^+\pi^+$ and $\pi^-\pi^-$ with fixed centrality $(30-80\%)$ with different k_T:



a) b)

Figure 18: Correlation function with consideration of track effects for centrality of $(30-80\%)$. a) $\pi^+\pi^+$ b) $\pi^-\pi^-$

Correlation functions for $\pi^+\pi^+$ and $\pi^-\pi^-$ with fixed k_T $(0.15 < k_T < 0.45)$ GeV/c with different centrality:



a) b)

Figure 19: Correlation function with consideration of track effects for k_T of $(0.15 < k_T < 0.45)$ GeV/c. a) $\pi^+\pi^+$ b) $\pi^-\pi^-$

4 Summary

In this work, we were able to study quark gluon plasma, using the fact, that in such conditions a correlation of particles' physical quantities, such as momentum and energies, occurs. We used data from the collisions of heavy ions Au+Au at energies of $\sqrt{s_{NN}} = 200$ GeV to extract necessary data for calculation of correlation functions for identically charged pions. At the end, we presented final correlation functions in different regions of centrality and pair's momentum with consideration of track splitting and track merging effects.

5 Acknowledgements

I would like to thank the University Center of JINR for giving me such an incredible opportunity to participate in the Winter Student Program and to broaden my horizon on Nuclear Physics.

I also would like to express my gratitude to my supervisor Alexey Aparin, who supported my journey throughout the program and gave me an opportunity to not only study such an amazing topic but also make great acquaintances with incredible people.

I'm also really grateful to Artem A. Korobitsin and Vinh Luong who were always willing to help me and explain difficult for understanding topics.

References

[1] Rendall C. Wells (2003) Azimuthal Dependence of Pion Interferometry in Au+Au Collisions at a Center of Mass Energy of $130 AGeV$.

[2] Mercedes López Noriega (2004) Pion Interferometry in AuAu Collisions at a Center of Mass Energy per Nucleon of 200 GeV.

[3] Michael Annan Lisa, Scott Pratt, Ron Soltz, Urs Wiedemann Femtoscopy in Relativistic Heavy Ion Collisions: Two Decades of Progress.

[4] Ulrich Heinz, Gregory Kestin Jozsó's Legacy: Chemical and Kinetic Freeze-out in Heavy-Ion Collisions

[5] https://root.cern.ch/