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Dzelepov Laboratory of Nuclear Problems

FINAL REPORT ON THE

SUMMER STUDENT PROGRAM

Introduction of sterile neutrino analysis in the GNA framework

Supervisors:

Dr. Dmitry Naumov

Konstantin Treskov

Maxim Gonchar

Student:

Arina Shaydurova

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Contents

Abstract

In this report the following topics are considered: neutrino oscillations, four-neutrino mixing, implementation 3+1 scheme in Global Neutrino Analysis (GNA) software.

There are analytical calculations:

1. A mixing matrix in four-neutrino case were received.
2. All sets of independent mass splittings were received. For each such set, other mass splittings were recalculated through this basis for two hierarchies: normal and inverted.

Result of these calculations were implemented in GNA framework as classes for mixing parameters, oscillation parameters and the probability of oscillations in four-neutrino case. Which can be used for creating models of experiments for analysis.

Introduction

Over the last 20 years a standard neutrino oscillation framework associated with small splitting between the neutrino mass states have become well-established. Beyond this model, several anomalies have been observed, one of them in reactor experiments.

Experiments show a 3σ deficit of neutrinos detected from reactors relative to the number predicted. There is a systematic difference between the experimental data and the theoretical data. This effect was named a *reactor neutrino anomaly*.

One of the ways to explain the anomaly is to add the fourth massive neutrino, affecting experiments through oscillation with active flavors $\{\nu_e, \nu_\mu, \nu_\tau\}$. This new massive type of neutrinos was named *sterile*. In general, number of sterile neutrinos is unknown, but for the explanation of the reactor anomaly at least one is required.

1 Neutrino oscillations

In the Standard Model of particle physics neutrinos are massless. However, experiments indicate that neutrinos not only have mass, but also are in the state of superposition of flavor eigenstates $\nu_\alpha,\ \alpha \in \{e,\ \mu,\ \tau,\ \ldots\}$. Flavor eigenstates are not identical to the mass eigenstates $\nu_k,\ k \in N$.

This superposition is carried out through an unitary mixing matrix U[1]

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle \,, \tag{1.1}$$

accordingly the massive states can be expressed in terms of flavor states

$$|\nu_k\rangle = \sum_\alpha U_{\alpha k} |\nu_\alpha\rangle \,, \tag{1.2}$$

and flavor states are orthonormal as well as massive states:

$$\langle \nu_\alpha | \nu_\beta \rangle = \delta_{\alpha\beta}, \qquad \langle \nu_k | \nu_j \rangle = \delta_{kj}. \tag{1.3}$$

As an evidence of mixing, neutrinos have been observed to change from one flavor to another during their propagation — a phenomenon called neutrino oscillations that was proposed in 1957 by Pontecorvo in an analogy to the K-meson oscillations.

The oscillations are generated by the interference of different massive neutrinos ν_k, which are produced and detected coherently because of their very small mass differences. More information about neutrino physics are in [1].

1.1 Neutrino oscillation probability

Because of mixing neutrino has a nonzero probability of transition from a state with flavor α to another state with flavor β — *transition probability*. In the case when $\alpha = \beta$ it is called *survival probability*.

[1] Mixing matrix U in 3-neutrino case called PMNS-matrix (Pontecorvo-Maki-Nakagawa-Sakata) and discussed in the section 1.2.

1.1.1 Transition probability

Probability of neutrino transition from α to β is written as

$$P_{\alpha\beta}(t) = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2, \tag{1.1.1.1}$$

and its depends on time (distance).

If the time evolution of equation (1.1) will be written implying that the massive neutrino states evolve in time as plane waves

$$|\nu_\alpha(t)\rangle = \sum_k U^*_{\alpha k}\, e^{-iE_k t}\, |\nu_k\rangle\,, \tag{1.1.1.2}$$

where $|\nu_k\rangle = |\nu_k(t=0)\rangle$ and $E_k = \sqrt{\mathbf{p}^2 + m_k^2}$, so the equation (1.1.1.1) can be revealed as

$$P_{\alpha\beta}(t) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\, e^{-i(E_k - E_j)t} \tag{1.1.1.3}$$

and can be simplified as

$$P_{\alpha\beta}(t) = \delta_{\alpha\beta} - 4\sum_{k>m} \mathrm{Re}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}) \sin^2\frac{(E_k - E_m)t}{2} + \\ +2\sum_{k>m} \mathrm{Im}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}) \sin(E_k t - E_m t). \tag{1.1.1.4}$$

Since the main part of the neutrinos is ultrarelativistic particles, it is a good approximation $E_k \approx E + m_k^2/(2E)$, where $E = |\mathbf{p}|$. Then

$$E_k - E_m \approx \frac{\Delta m^2_{km}}{2E}, \quad \text{where } \Delta m^2_{km} = m_k^2 - m_m^2. \tag{1.1.1.5}$$

Transition probability with (1.1.1.5) and with approximation $t \approx L$, where L is a baseline, in the more useful form

$$P_{\alpha\beta}(L, E) \approx \delta_{\alpha\beta} - 4\sum_{k>m} \mathrm{Re}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}) \sin^2\Big(\frac{\Delta m^2_{km} L}{4E}\Big) + \\ +2\sum_{k>m} \mathrm{Im}(U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}) \sin\Big(\frac{\Delta m^2_{km} L}{2E}\Big). \tag{1.1.1.6}$$

The transition probability of antineutrino oscillations can be obtained by conjugating the expression (1.1.1.6).

1.1.2 Survival probability

When $\alpha = \beta$, the equation (1.1.1.6) becomes simpler

$$P_{\alpha\alpha}(L,E) \approx 1 - 4\sum_{k>m} |U_{\alpha k}|^2 |U_{\alpha m}|^2 \sin^2\Big(\frac{\Delta m^2_{km} L}{4E}\Big). \qquad (1.1.2.1)$$

This type of oscillation probabilities is very important and means probability that the neutrino or antineutrino[2] will not change its flavor during its propagation. Only this type of probability will be used further in this work.

1.2 Standard theory of neutrino oscillations

In the standard theory there are only three known types of the flavor neutrino. They are *active* with $\alpha \in \{e,\, \mu,\, \tau\}$. Consequently there are three massive types of neutrino $\nu_i,\ i \in \{0, 2, 3\}$ and the equation (1.1) is expressed as:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 3} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \qquad (1.2.1)$$

in this case the mixing matrix U is a Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which has an explicit form:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}, \qquad (1.2.2)$$

where $c_{ij} = cos(\theta_{ij}),\ s_{ij} = sin(\theta_{ij})$, and δ_{CP} is a CP-violating phase, θ_{ij} are the mixing angles.

As seen in (1.2.2), PMNS-matrix has size $[3 \times 3]$, because the number of flavors and massive states is $N = 3$. It is parametrized by three mixing angles (number of angles $= N(N-1)/2$) and one complex phase (number of complex phases $= 1 + N(N-3)/2$).

In case of more flavor states, for example, four, 6 mixing angles and 3 complex phases are required. But why do it need four flavor states? In the standard theory there are only three active flavors.

[2] They have the same formula of survival probability.

2 Reactor anomaly

In 2010 it turned out that all reactor experiments observe a flux of about 5% less than predicted by the theory. This effect was called a reactor anomaly [2]. There are different ways to explain it. One of them is to add a new type of neutrinos. This new fourth nonstandard heavy type is not expected to interact and therefore is named sterile.

2.1 Sterile neutrinos

"Sterile" means no standard model interactions. Active neutrinos $\{\nu_e, \nu_\mu, \nu_\tau\}$ can oscillate into sterile neutrinos ν_s. A new type of flavor and a new type of massive states are introduced into mixing, but it is unknown how many sterile neutrino states may exists.

In Fig.2.1.1. one can see a simple scheme of neutrino bases for mixing.

Mass basis:	ν_1	ν_2	ν_3	ν_4	ν_5	$\cdots$
Flavor basis:	ν_e	ν_μ	ν_τ	ν_{s_1}	ν_{s_2}	$\cdots$
		ACTIVE			STERILE	

Figure 2.1.1: Scheme of neutrino bases for mixing including sterile neutrinos.

In order to study reactor anomaly we expand the neutrino oscillation theory in GNA[3] framework by adding new type of flavors — sterile.

[3] Global Neutrino Analysis.

3 Four-neutrino mixing

The number of sterile neutrino types (should they exist) is not yet theoretically established. For simplicity our analysis is restricted to the 3+1 four-neutrino scheme.

3.1 Mixing matrix

A full 3+1 model has a mixing matrix with size $[4 \times 4]$ $(N = 4)$ that connects all three active plus single sterile flavor states with four mass states:

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ U_{21} & U_{22} & U_{23} & U_{24} \\ U_{31} & U_{33} & U_{33} & U_{34} \\ U_{41} & U_{42} & U_{43} & U_{44} \end{pmatrix}. \tag{3.1.1}$$

In the case of only one sterile neutrino, U is typically, as in [3], parameterized by

$$U = R_{34}\, \tilde{R}_{24}\, \tilde{R}_{14}\, R_{23}\, \tilde{R}_{13}\, R_{12}, \tag{3.1.2}$$

where the matrix R_{ij} is a rotation by the angle θ_{ij} in the corresponding ij space, for example,

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 \\ 0 & -s_{23} & c_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ or } \tilde{R}_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix}, \tag{3.1.3}$$

In this parametrization we found the explicit form of the 3+1 mixing matrix. The rows element-by-element are written below.

The first row $(1 \rightleftarrows e)$:

$$U_{11} = c_{12}\, c_{13}\, c_{14}, \quad U_{12} = s_{12}\, c_{13}\, c_{14}, \quad U_{13} = c_{14}\, s_{13} e^{-i\delta_{13}}, \quad U_{14} = s_{14} e^{-i\delta_{14}}; \tag{3.1.4}$$

the second row $(2 \rightleftarrows \mu)$:

$$U_{21} = -c_{12}(c_{13}\, s_{14} e^{i\delta_{14}}\, s_{24} e^{-i\delta_{24}} + c_{24}\, s_{23}\, s_{13} e^{i\delta_{13}}) - c_{23}\, c_{24}\, s_{12}, \tag{3.1.5}$$

$$U_{22} = -s_{12}(c_{13}\, s_{14} e^{i\delta_{14}}\, s_{24} e^{-i\delta_{24}} + c_{24}\, s_{23}\, s_{13} e^{i\delta_{13}}) + c_{12}\, c_{23}\, c_{24}, \tag{3.1.6}$$

$$U_{23} = c_{13}\,c_{24}\,s_{23} - s_{13}e^{-i\delta_{13}}\,s_{14}e^{i\delta_{14}}\,s_{24}e^{-i\delta_{24}}, \tag{3.1.7}$$

$$U_{24} = c_{14}\,s_{24}e^{-i\delta_{24}}; \tag{3.1.8}$$

the third row ($3 \rightleftarrows \tau$):

$$U_{31} = -c_{12}\big(c_{13}\,c_{24}\,s_{34}\,s_{14}e^{i\delta_{14}} - s_{13}e^{i\delta_{13}}(s_{23}\,s_{34}\,s_{24}e^{i\delta_{24}} - c_{23}\,c_{34})\big) + \\ + s_{12}(c_{23}\,s_{34}s_{24}e^{i\delta_{24}} + s_{23}\,c_{34}), \tag{3.1.9}$$

$$U_{32} = -s_{12}\big(c_{13}\,c_{24}\,s_{34}\,s_{14}e^{i\delta_{14}} - s_{13}e^{i\delta_{13}}(s_{23}\,s_{34}\,s_{24}e^{i\delta_{24}} - c_{23}\,c_{34})\big) - \\ - c_{12}(c_{23}\,s_{34}s_{24}e^{i\delta_{24}} + s_{23}\,c_{34}), \tag{3.1.10}$$

$$U_{33} = -c_{24}\,s_{34}\,s_{14}e^{i\delta_{14}}\,s_{13}e^{-i\delta_{13}} - c_{13}(s_{23}\,s_{34}\,s_{24}e^{i\delta_{24}} - c_{23}\,c_{34}), \tag{3.1.11}$$

$$U_{34} = c_{14}\,c_{24}\,s_{34}; \tag{3.1.12}$$

the fourth row ($4 \rightleftarrows s$):

$$U_{41} = -c_{12}\big(c_{13}\,c_{24}\,c_{34}\,s_{14}e^{i\delta_{14}} - s_{13}e^{i\delta_{13}}(s_{23}\,c_{34}\,s_{24}e^{i\delta_{24}} + c_{23}\,s_{34})\big) + \\ + s_{12}(c_{23}\,c_{34}s_{24}e^{i\delta_{24}} - s_{23}\,s_{34}), \tag{3.1.13}$$

$$U_{42} = -s_{12}\big(c_{13}\,c_{24}\,c_{34}\,s_{14}e^{i\delta_{14}} - s_{13}e^{i\delta_{13}}(s_{23}\,c_{34}\,s_{24}e^{i\delta_{24}} + c_{23}\,s_{34})\big) - \\ - c_{12}(c_{23}\,c_{34}s_{24}e^{i\delta_{24}} - s_{23}\,s_{34}), \tag{3.1.14}$$

$$U_{43} = -c_{24}\,c_{34}\,s_{14}e^{i\delta_{14}}\,s_{13}e^{-i\delta_{13}} - c_{13}(s_{23}\,c_{34}\,s_{24}e^{i\delta_{24}} + c_{23}\,s_{34}), \tag{3.1.15}$$

$$U_{44} = c_{14}\,c_{24}\,c_{34}; \tag{3.1.16}$$

There are 6 mixing angles θ_q, $q \in \{12, 13, 14, 23, 24, 34\}$ and 3 complex phases δ_p, $p \in \{13, 24, 34\}$.

In the reactor experiments only electron antineutrinos can be detected. Accordingly, it is not possible to observe the transition of one type of neutrino to another. It means in our analysis it needs only the first row of 3+1 mixing matrix to derive the survival probability of electron antineutrinos.

3.2 Survival probability of electron antineutrinos

The survival probability of electron antineutrinos can be written as

$$P_{ee} = 1 - 4\sum_{k>m} |U_{1k}|^2 |U_{1m}|^2 \sin^2\Big(\frac{\Delta m^2_{km} L}{4E}\Big), \tag{3.2.1}$$

with $k \in \{4,3,2\}$, $m \in \{1,2,3\}$. Using the first row (3.1.4) of mixing matrix, the survival probability (3.2.1) can be rewritten as

$$\begin{aligned} P_{ee} = 1 - c^4_{14} s^2_{12} \sin^2(2\theta_{13}) \sin^2\Delta_{32} - c^4_{14} c^2_{12} \sin^2(2\theta_{13}) \sin^2\Delta_{31} - \\ - c^4_{14} c^4_{13} \sin^2(2\theta_{12}) \sin^2\Delta_{21} - s^2_{13} \sin^2(2\theta_{14}) \sin^2\Delta_{43} - \\ - c^2_{13} s^2_{12} \sin^2(2\theta_{14}) \sin^2\Delta_{42} - c^2_{13} c^2_{12} \sin^2(2\theta_{14}) \sin^2\Delta_{41}, \end{aligned} \tag{3.2.2}$$

where $\Delta_{km} = \Delta m^2_{km} L/4E$ denote the oscillation phases.

The following approximations may be done in order to simplify (3.2.2): the Δ_{43} mode is further suppressed by both θ_{13} and θ_{14}, and can therefore be safely neglected; take in the limit $c^2_{13} = c^2_{14} = 1$; because of relatively short L in the reactor experiments, Δ_{21} can be neglected; $\Delta_{32} \simeq \Delta_{31} = \Delta_{ee}$ and $\Delta_{42} \simeq \Delta_{41}$:

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2\Delta_{ee} - \sin^2 2\theta_{14} \sin^2\Delta_{41}, \tag{3.2.3}$$

this expression is used in [3] and [4].

In experiments some parameters of three-neutrino mixing were measured or estimated: $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, Δm^2_{21}, $|\Delta m^2_{32}|$ may be found in PDG [5]; also θ_{13} and Δm^2_{32} are in [6]. But in P_{ee} (3.2.3) there are new parameters: $\sin^2\theta_{14}$ (read θ_{14}) and Δ_{41} (read Δm^2_{41}). Their current values are unknown and expected to be $\Delta m^2_{41} \sim 1\,\mathrm{eV}^2$ and $\sin^2 2\theta_{14} \sim 0.14$ in order to explain the reactor anomaly.

4 GNA

GNA (Global Neutrino Analysis) is the software for analyzing data of neutrino experiments. Currently it is devoted to the analysis JUNO and Daya Bay. It is designed with having joint analysis of various experiments in mind.

GNA framework consists of separate modules that can be combined into the computational block. The calculation result of each module is cached and recalculated only when it is really needed, i.e. when the parameters or inputs of these modules are modified.

The task of an user is to use those blocks as ingredient to construct a computational graph producing the theoretical predictions and statistic.

The user interface of GNA is implemented in Python. Data analysis algorithms are implemented in C++. The interface is linked with C++ backend via PyROOT.

4.1 Implementation of 3+1 scheme

For implementation formulas from 3+1 scheme in GNA a mass hierarchy should be chosen as well and a set of independent mass splittings.

4.1.1 Neutrino mass

The neutrino oscillations probabilities depend on mass splittings; in the case of 3-neutrino mixing, there are two independent splittings. In the case with 4-neutrino mixing, there are three independent splittings out of the 6 existing. There are also hierarchies of these masses.

Normal (NO) and *inverted* (IO) hierarchies are used in GNA. They are distinguished by a value of $\alpha = \{1, -1\}$ respectively. It is unknown, how much m_4 weighs relative other masses $\{m_3, m_2, m_1\}$, so we supposed m_4 is the heaviest. It is shown in Fig.4.1.1.

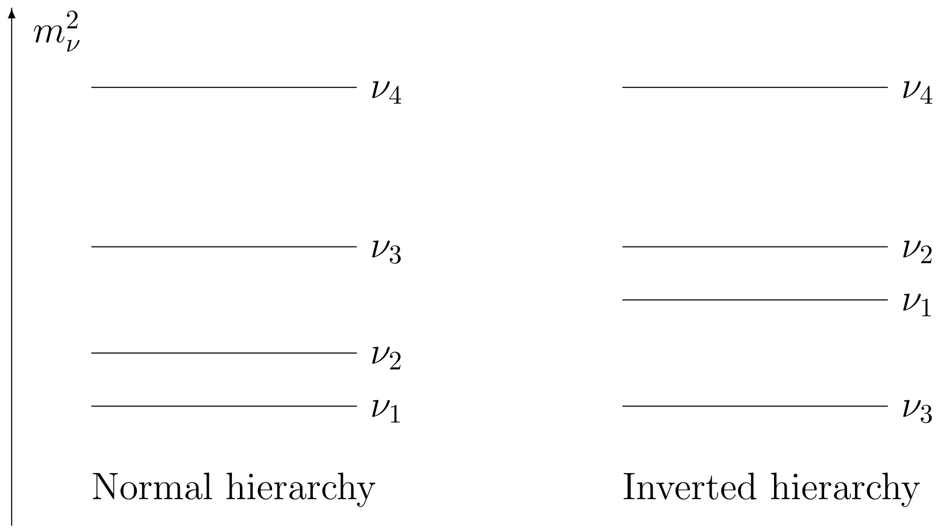

Figure 4.1.1.1: Graphical representation between the mass neutrino states. The ordinate position corresponds to the absolute square value neutrino mass (not in a real scale) for normal and inverted mass hierarchies.

The user can choose a basis — an independent set of mass splittings. Six such sets were provided, which are given in the Table 4.1.1.1

Table 4.1.1.1: Sets of independent mass splittings.

Number	Set
1	Δm_{21}^2, Δm_{31}^2, Δm_{41}^2
2	Δm_{21}^2, Δm_{32}^2, Δm_{41}^2
3	Δm_{21}^2, Δm_{31}^2, Δm_{42}^2
4	Δm_{21}^2, Δm_{32}^2, Δm_{42}^2
5	Δm_{21}^2, Δm_{31}^2, Δm_{43}^2
6	Δm_{21}^2, Δm_{32}^2, Δm_{43}^2

Δm_{21}^2 was chosen as independent in each of the 6 sets, because its value and sign is well measured in solar neutrino experiments: $\Delta m_{21}^2 \simeq +(7.59 \pm 0.21) \cdot 10^{-5}\text{eV}^2$.

For example, if the user want to use the first basis, i.e. the mass splittings with indices {21, 31, 41}, other squared mass differences {32, 42, 43} will be recalculated as

$$\begin{aligned}\Delta m^2_{32} &= \Delta m^2_{31} - \alpha\Delta m^2_{21},\\ \Delta m^2_{42} &= \Delta m^2_{41} - \Delta m^2_{21},\\ \Delta m^2_{43} &= \Delta m^2_{41} - \alpha\Delta m^2_{31}.\end{aligned} \tag{4.1.1.1}$$

All of this sets are implemented in GNA by us and available to the user, expressions as (4.1.1.1) are universal for our hierarchies by α value. In the code the normal hierarchy is default, but the user can change the $\alpha = 1$ to $\alpha = -1$.

4.1.2 Electron-neutrino survival probability

Within GNA the classes for mixing parameters, oscillation parameters, and the probability of oscillations in four-neutrino case were implemented in the C++.

Using these classes, we programmed the formula of electron-neutrino survival probability in four-neutrino case (3.2.3) in the following form

$$P(E_{\bar{\nu}}) = \sum_j w_j(\theta) P_j(\Delta m^2, E_{\bar{\nu}}), \tag{4.1.2.1}$$

where the mixing angle dependent weights w_j and mass splittings dependent oscillatory terms P_j are factorized. Since only the oscillatory terms are energy dependent, when doing fits it's more computationally efficient to take the weights out of the integrals, making the recomputations a lot faster in case only mixing angles are modified.

All the unknown parameters in the code are free and should be initialized by the user. Measured values for mixing angles and mass splittings are taken from [5] (for 2016 data). For example, the result of this work with specific selected values of unknown parameters is shown in Fig.4.1.2.1.

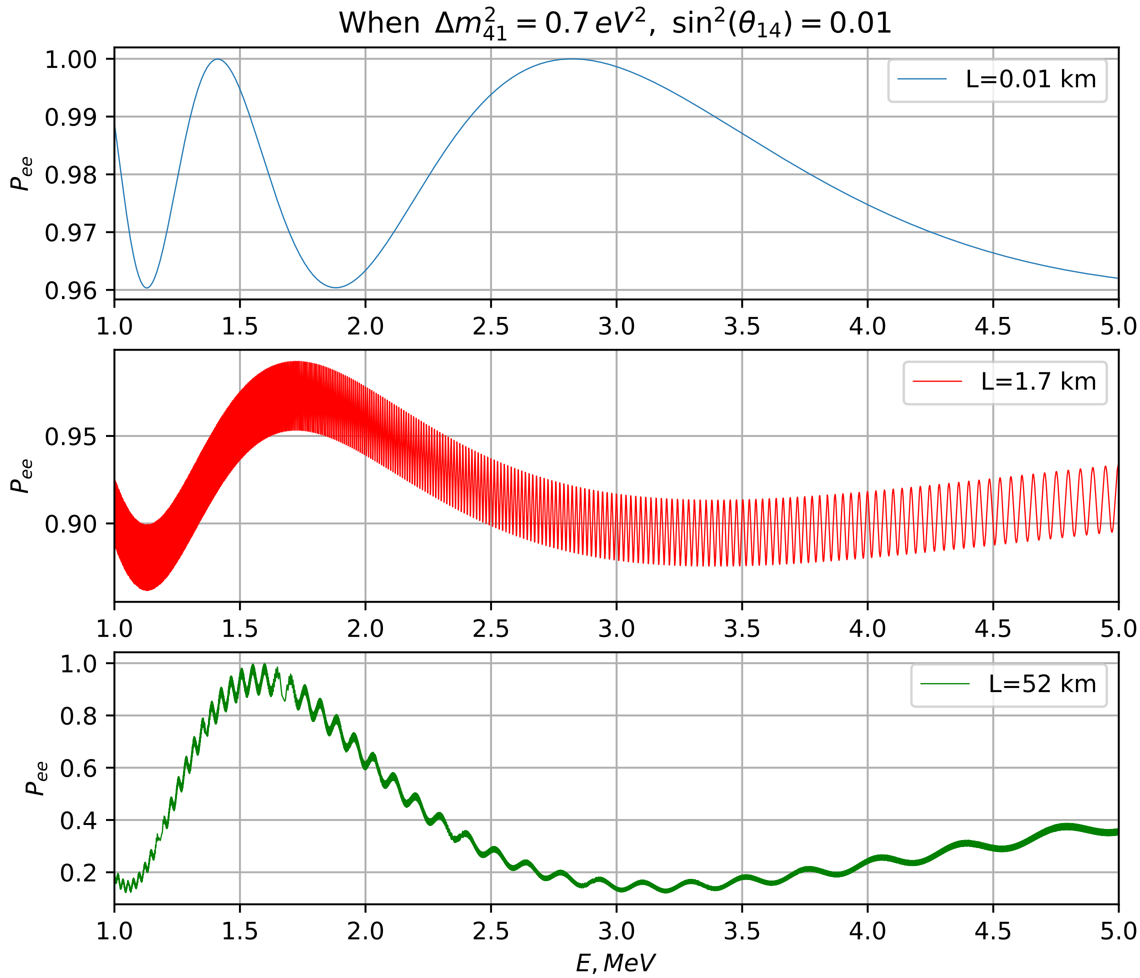

Figure 4.1.2.1: Survival probability for electron neutrino $P_{e\to e} \equiv P_{ee}$ depending on the neutrino energy E with example values of unknown parameters $\Delta m^2_{41} = 0.7\,\mathrm{eV}^2$ and $\sin^2\theta_{14} = 0.01$, for three different baselines L.

Also the newly implemented classes are suitable for any other survival probabilities $P_{\alpha\beta}$, i.e. not only for P_{ee}. In case the transition probability is needed the class should be completed by the imaginary part in formula (1.1.1.6).

4.2 Preliminary model of the reactor neutrino experiment

In this model the number of events in each bin of visible energy can be found by the implementation a following general formula

$$N_i = \int\limits_{E_i}^{E_{i+1}} dE \, \frac{dE_{\bar{\nu}}}{dE} \, \sigma(E_{\bar{\nu}}) \, P_{ee}(E_{\bar{\nu}}) \sum_k n_k S_k(E_{\bar{\nu}}), \qquad (4.2.1)$$

where N_i is the event number in the i-th bin containing events with energies $[E_i, E_{i+1}]$; E and $E_{\bar{\nu}}$ are positron (visible energy) and neutrino energies; $\sigma(E_{\bar{\nu}})$ is the IBD cross section; $P_{ee}(E_{\bar{\nu}})$ is the oscillation probability (in our case for 4-neutrino mixing); $S_k(E_{\bar{\nu}})$ is the antineutrino spectrum of the k-th isotope with corresponding normalization n_k.

The given formula are only for the case of one reactor. If there are several of them, an additional summation inside the integral should be performed.

Due to the limited time of the Summer Student program, the model was implemented only partially, without the oscillation probability.

A computation of 1-dimesional integral with not oscillated flux

$$N_i = \int\limits_{E_i}^{E_{i+1}} dE \, \frac{dE_{\nu}}{dE} \, \sigma(E_{\nu}(E)) \, \sum_k n_k \, S_k(E_{\bar{\nu}}(E)) \, , \qquad (4.2.2)$$

where index k in antineutrino spectrum in our model is used for four isotopes: ^{238}U, ^{235}U, ^{239}Pu, ^{241}Pu.

The result of computational (4.2.2), is shown in Fig.4.2.1.

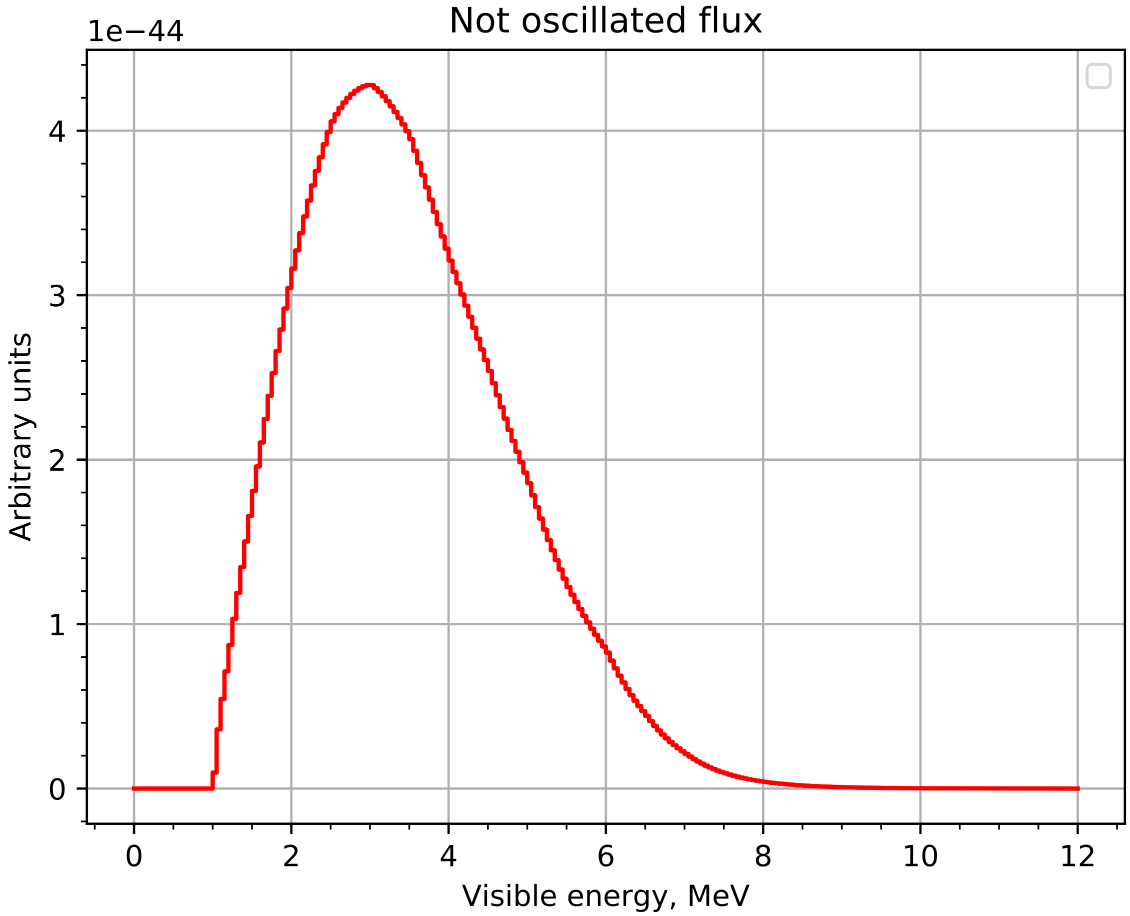

Figure 4.2.1: Integrated reactor electron antineutrino flux without oscillations as a function visible energy.

The computational process in GNA is a sequence of transformations that can be shown in graphs. The computational graph of (4.2.2) is in Fig.4.2.2.

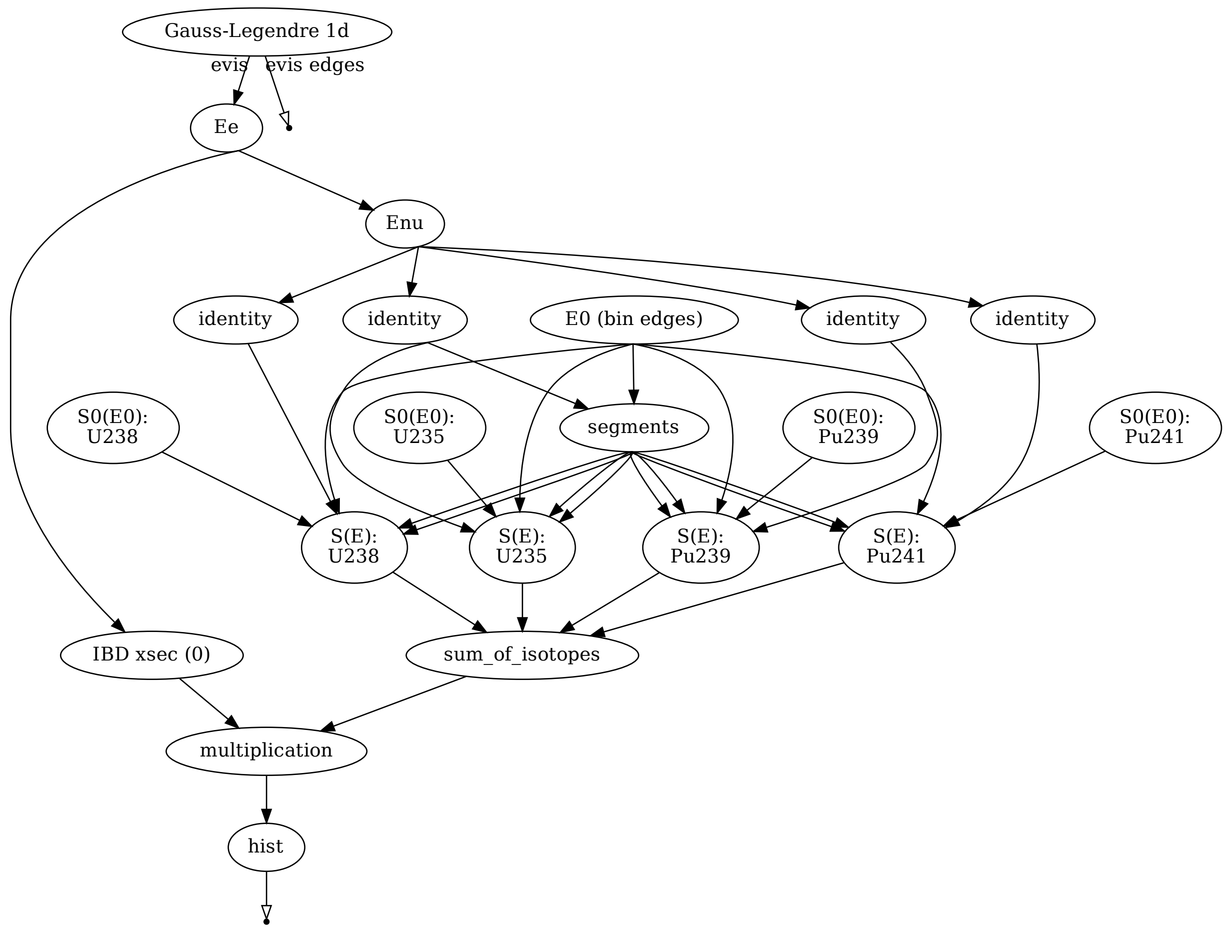

Figure 4.2.2: real graph of calculation of the integral (in the form of a histogram) of a not oscillated flux with four isotopes: ^{238}U, ^{235}U, ^{239}Pu, ^{241}Pu; where evis is visible energy, which is provided by the integrator (Gauss-Legender 1d). These are all the points needed to compute the integrals for each bin of the output histogram, Ee is a positron energy provided by cross section (IBD xsec (0)), which is computed from evis, Enu is neutrino energy provided by cross section, which is computed from Ee.

In order to complete the model one need to multiply the integrand of (3.2.2) by the survival probability P_{ee}.

Conclusion

The following steps were taken in this work to introduce to sterile neutrino analysis in the GNA framework:

- For case with mixing three active plus single sterile flavors the explicit form of mixing matrix U was received.
- All sets of independent mass splittings (6 sets) were implemented. For each such set, other mass splittings were recalculated through this basis for two hierarchies: NO and IO (18 equations). These equations are written in the universal form for NO and IO.
- Within the GNA framework the classes for mixing parameters, oscillation parameters and the probability of oscillations in four-neutrino case were implemented.
- We created a python module to plot the electron-neutrino survival probability which uses our classes.
- We created a python module to compute the visible spectrum without oscillations and plot it in a preliminary model of the reactor neutrino experiment.

The further work includes the finalizing of a model by adding the antineutrino survival probability.

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