JOINT INSTITUTE FOR NUCLEAR RESEARCH
Bogoliubov Laboratory of Theoretical Physics

FINAL REPORT ON THE SUMMER STUDENT PROGRAM

Evolution equations solution in QED

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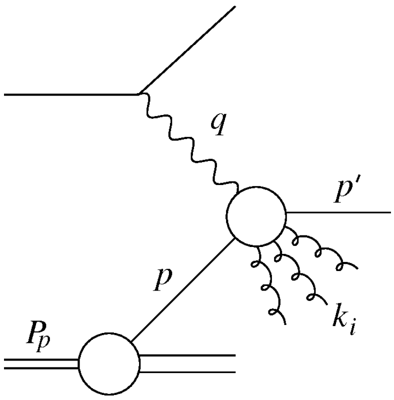
Contents

1 Abstract

In the present report we calculated third order QED corrections $O(\alpha^3)$ in the next-to-leading logarithmic order (NLO) of electron parton distribution function (PDF) in a hard scattering process. These corrections were computed using perturbative approach as a result of iterative solution of integral Dokshitzer–Gribov–Lipatov–Altarelli–Parisi evolution equations (GDLAP)in QED.

2 Introduction

In modern high energy physics and elementary particle physics are using concept of parton function distribution (PDF) when calculating different processes. Initially concept of parton distribution function appeared in so-called parton model of proton in purpose to take into account contributions radiative corrections to deep inelastic scattering (DIS) process.



Deep Inelastic Scattering

To take into account huge number of radiative corrections on different momentum scales are used Dokshitzer–Gribov–Lipatov–Altarelli–Parisi evolution equations (GDLAP). DGLAP equations is an analog renormalization group equation or Callan–Symanzik equation and using in processes in quantum chromodynamics (QCD).

DGLAP evolution equation:

$$\frac{\partial f_{ij}(x,Q^2)}{\partial Ln(Q^2)} = \frac{\alpha_s(Q^2)}{2\pi} P_{ji} \otimes f_{ij}$$

where f_{ij} is PDF of parton, Q^2 is a scale of transverse impulses of partons, x is a part of longitudinal momentum, $\alpha_s(Q^2)$ is running a coupling constant in QCD and P_{ji} is a splitting function corresponding transition parton i to parton j.

Convolution integral:

$$(A(x) \otimes B(x))(z) = \int_0^1 \int_0^1 A(x)B(y)\delta(z-xy)dxdy = \int_z^1 \frac{dx}{x} A(z)B(\frac{z}{x})$$

Solutions of this evolution equations can be found using iteration method:

$$D_{ij} = D_{ij}^{(0)} + \int_{m^2}^{Q^2} \frac{ds}{s} \frac{\alpha_s(s)}{2\pi} P_{ji} \otimes D_{ij} = D_{ij}^{(0)} + \int_{m^2}^{Q^2} \frac{ds}{s} \frac{\alpha_s(s)}{2\pi} \int_x^1 \frac{dz}{z} P_{ji}(z) D_{ij}(\frac{x}{z})$$

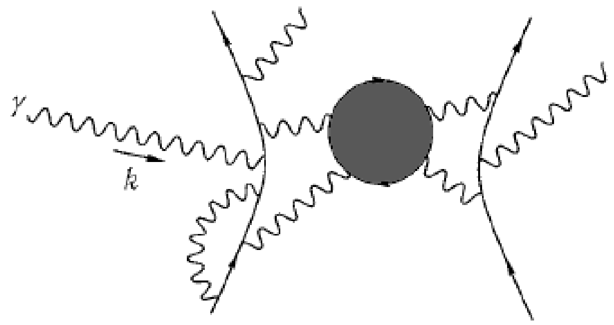
where D_{ij} is a PDF.

3 Our purpose

Since the parton model can be applied to processes in quantum electrodynamics (QED). Our purpose consist in applying method of iteration solution of evolution equation for searching electron's PDF and check this results in articles [1] and [2]. We compared our result for cross section of electron scattering in third order of α in NLO $O(\alpha^3 L^2)$ approximation, represented by formula:

$$\sigma^{NLO} = D_{ie} \otimes \sigma^h_{ij} \otimes D_{ej}$$

where σ^h_{ij} is a cross section of a hard subprocess in the electron scattering process, represented by grey circle on picture below. Photon lines mean radiative corrections to electron movement, which contribution we calculated.



Methods

We used QED evolution equation in integral form to calculate electron's PDF:

$$\begin{cases} D_{ee} = D_{ee}^{(0)} + \displaystyle\int_{m_e}^{Q^2} \frac{ds}{s}\frac{\alpha(s)}{2\pi}\left(P_{ee}\otimes D_{ee} + P_{e\gamma}\otimes D_{\gamma e}\right) \\ \\ D_{\gamma e} = D_{\gamma e}^{(0)} + \displaystyle\int_{m_e}^{Q^2} \frac{ds}{s}\frac{\alpha(s)}{2\pi}\left(P_{\gamma\gamma}\otimes D_{\gamma e} + P_{\gamma e}\otimes D_{ee}\right) \end{cases}$$

With initial conditions:

$$\begin{cases} D_{ee}^{(0)}(x) = \delta(1-x) + \dfrac{\alpha}{2\pi}d_1^{ee}(x) \\ \\ D_{\gamma e}^{(0)}(x) = \dfrac{\alpha}{2\pi}d_1^{\gamma e}(x) \end{cases}$$

And note that $\alpha(s)$ is running in $\bar{M}S$ scheme coupling constant:

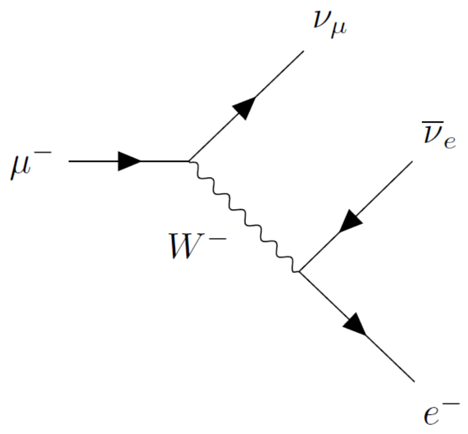
$$\frac{\alpha(s)}{2\pi} = \frac{\alpha}{2\pi}\left(1 + \frac{2}{3}\frac{\alpha}{2\pi}L + \frac{13}{27}(\frac{\alpha}{2\pi})^2 L + ...\right)$$

We used FORM for calculations electron's PDF and electrons scattering cross section σ^{NLO}

```
* Iterative solution for D_ee in LLA (NLA)
S L,A,A1,o;
S DZ3;
S delta,Pee0,dee1,Pee1,Pge0,dge1,Pge1,Pgg0,Pgg1,Peg0,Peg1;
nwrite statistics;

* running alpha:
* A = \alpha(0)/(2\pi),  Al2pi = \alpha(Q^2)/(2\pi) in MSbar

G Al2pi = A*(1 + 2/3*A*L - 10/9*A - A^2*13/27*L + 4*A^2*DZ3 + 4/9*A^2*L^2);

* L is the large log

L Pee = Pee0 + Al2pi*Pee1;
L Pge = Pge0 + Al2pi*Pge1;
L Pgg = Pgg0 + Al2pi*Pgg1;
L Peg = Peg0 + Al2pi*Peg1;

* The initial condition for interations:

G Dee0 = delta + A*dee1;
L Dge0 = 0 + A*dge1;
*G Dee0 = delta + Al2pi*dee1;
*L Dge0 = 0 + Al2pi*dge1;

* the first iteration

L Dee1 = Dee0 + o*Al2pi*(Pee*Dee0+Peg*Dge0);
L Dge1 = Dge0 + o*Al2pi*(Pgg*Dge0+Pge*Dee0);
.sort
* integration over s-prime
ID o*L^3 = L^4/4;
ID o*L^2 = L^3/3;
ID o*L   = L^2/2;
ID o     = L;
.sort


* the second iteration

L Dee2 = Dee0 + o*Al2pi*(Pee*Dee1+Peg*Dge1);
L Dge2 = Dge0 + o*Al2pi*(Pgg*Dge1+Pge*Dee1);

.sort
*B A,L;
*print Dee2;
*.end

* integration over s-prime
ID o*L^3 = L^4/4;
ID o*L^2 = L^3/3;
ID o*L   = L^2/2;
ID o     = L;
.sort
skip Al2pi,Dee0;
ID delta = 1;
ID A^3 = 0;
.sort
B A,L;
print Dee2,Dge2;
.sort
*.end

* the third iteration

G Dee3 = Dee0 + o*Al2pi*(Pee*Dee2+Peg*Dge2);
G Dge3 = Dge0 + o*Al2pi*(Pgg*Dge2+Pge*Dee2);
.sort

* integration over s-prime
ID o*L^3 = L^4/4;
ID o*L^2 = L^3/3;
ID o*L   = L^2/2;
```

4 Result

We calculated total cross section considering radiative corrections up to third order of perturbation theory in NLO and checked our result with the same result but with another calculation in articles [1] and [2]. And after that we plan to apply our calculations into muon decay process. Based on article [3] we improve accuracy of electron's PDF.



Muon weak decay

5 Acknowledgments

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6 References

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[2] J. Ablinger, J. Blumlein, A. D. Freitas, K. Schonwald "Subleading logarithmic QED initial state corrections to $e^+e^- \to \gamma^*/Z^{0*}$ to $O(\alpha^6 L^5)$" Nucl.Phys.B 955 (2020) 115045

[3] A. Arbuzov, K. Melnikov "$O(\alpha^2 Ln(\frac{m_\mu}{m_e}))$ corrections to electron energy spectrum in muon decay" Phys.Rev.D 66 (2002) 093003