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Primary and secondary vertices reconstruction in the BM@N experiment

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Abstract

This report discusses the issue of vertex finding at BM@N experiment, which aims at studying collisions of the elementary particles and ions with a fixed target. There were different methods implemented to found the vertices. For the primary vertex finding, tracks, which were near to the target, were approximated by straight lines. Also it was the analysis of efficiency of primary vertex finding realized. For the secondary vertices, tracks were approximated in pairs by splines or polynomials near to point of the closest approach. After the calculation of minimum distance between two tracks, some of the couples, which answered the criteria, were put to the Armenteros-Podolanski plane. As a result, primary and secondary vertices were found and identification of λ^0-hyperons and their decay products was done.

Introduction

Nowadays one of the most interesting issues of high energy physics is heavy ion collisions, which provide a unique opportunity to study the nuclear matter under extreme density and temperature. However, such a unique problem requires special methods and conditions for solution. Therefore, it was a BM@N (part of NICA experiment) built [1]. The BM@N experiment, short from Baryonic Matter at the Nuclotron, is a set-up, which aims at studying collisions of the elementary particles and ions with a fixed target at energies (laboratory system) up to 4 GeV per nucleon (for Au^{79+}). The main tracker of the BM@N is GEM detector, that contains 12 stations, which are orthogonal to z axis [2]. Today, the BM@N is still under construction, and one of the several tasks to do is vertex finding.

There are two types of vertices which will be investigated. Primary vertex is produced by heavy ion collisions with a fixed target, and it is possible to maintain that primary vertex is the "start point" of the further experiment. So, it is very important to find this point [3], [4]. Another kind of vertex is secondary vertex. Usually, there are many particles, which are produced in the primary vertex and decay between the target and detectors (for instance, Λ^0-hyperons). As a result, these particles and their vertices may provide significant information about fusions, etc [5].

One of the feature of the task is a lack of information about types of particles, i.e. mass, charge and other native characteristics of particle are unknown before the start of vertices finding algorithm. This fact, which is caused by characteristics of GEM-detector stations, influenced the development of the algorithm.

Algorithm of primary vertex finding

Description of the algorithm

The primary vertex is a point, where a high energy beam collides with a matter of target. After the collision, there are many particles fly via the GEM-detectors. Thus, we get only spatial coordinates of hits, without any other characteristics. Therefore, the primary finding algorithm has to have a few steps, each of them provides more information about track.

First of all, several criteria are used to find the hits, which were probably produced by the same particle. After that, hits, which belong to the same particle, form the track. Then, tracks are propagated to the area of primary vertex by Kalman filter [6]. This stage of algorithm was being already implemented by BM@N software group [7].

Next, we request the prediction of Kalman filter on $z = -1; 1$ planes (it is assumed that the vertex located on $z = 0$ plane). Thus, it is possible to construct the directing vector $\vec{p_1}$ of the straight line, which approximates the track. Also, we need to choose the parametrisation. Taking into consideration a geometry of BM@N and bijection between the point of track and z coordinate, we may select the $t = z$ parametrization. So, then we can write the system of equations, where $\vec{r}$ is a vector of distance between the primary vertex (radius-vector $\vec{\omega}$) and the straight line:

$$\vec{p_0} + \vec{p_1} t_1 = \vec{\theta_0} \tag{1}$$

$$\vec{r} = \vec{\omega} - \vec{\theta_0} \tag{2}$$

$$\vec{p_1} \cdot \vec{r} = 0 \tag{3}$$

As a result, we have 4 (let us assume that $\vec{\omega}$ is known) unknown variables (r_x, r_y, r_z, t_1) and 4 linear equations, so it is possible to write $\vec{r}$ as a function $\vec{r} = f(\vec{\omega}, \vec{p_0}, \vec{p_1})$.

However, we consider, for instance, n straight lines, therefore it is needed to minimize the function $F = \sum_{i=0}^{n} |\vec{r_i}|$. So, it is enough to examine $dF = 0$. Moreover, we can investigate 3 equations instead of the previous one due to independence of variables $(\omega_x, \omega_y, \omega_z)$:

$$\frac{\partial F}{\partial \omega_x} = 0 \quad \frac{\partial F}{\partial \omega_y} = 0 \quad \frac{\partial F}{\partial \omega_z} = 0 \tag{4}$$

This linear system has an analytical solution. Finally, we solve the system and find the coordinates $\vec{\omega}$ of the vertex.

Analysis of efficiency

Not all of tracks are equally good for primary vertex finding, therefore an analysis of efficiency was realized. There were a few criteria, which were developed to separate the best tracks for finding algorithm. All of benchmarks depend on characteristics of track and will be discussed further.

One of the significant parameters is number of GEM stations, that particles pass through. We divided all of tracks into groups (4 stations, 5-6, 7-9, 10-11, 12) and find the primary vertex for all of possible combinations ($2^5 = 32$). There are only primary protons were used for the investigation. Results of the test are showed on the fig.1-6, where $\mathbb{T}$ is a space of transposition (for example, $t = 15_{10} = 01111_2$ is a combination without only first group), σ is a RMS (we used Gauss function 5 for fitting the results), and μ is a mathematical expectation.

$$f(x) = p_0 e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{5}$$

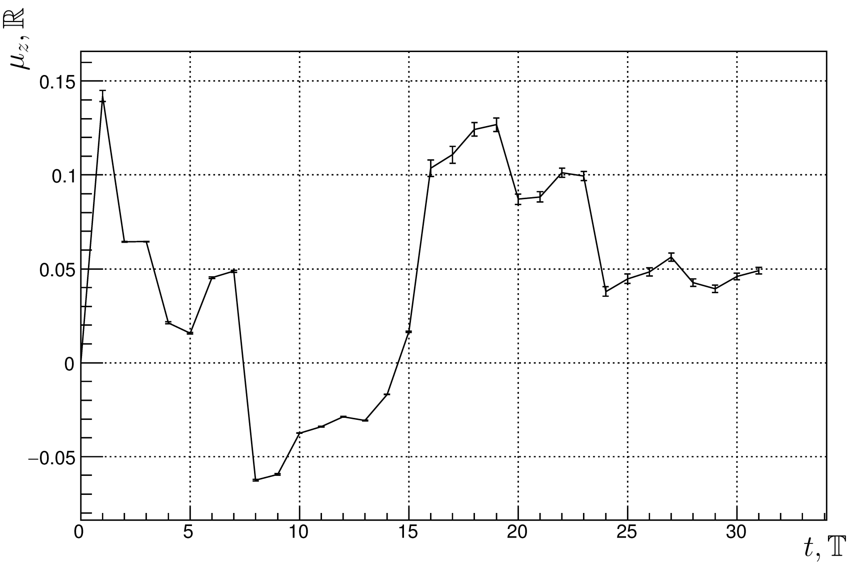

Figure 1: Mathematical expectation $\mu_x(t)$ of different combinations

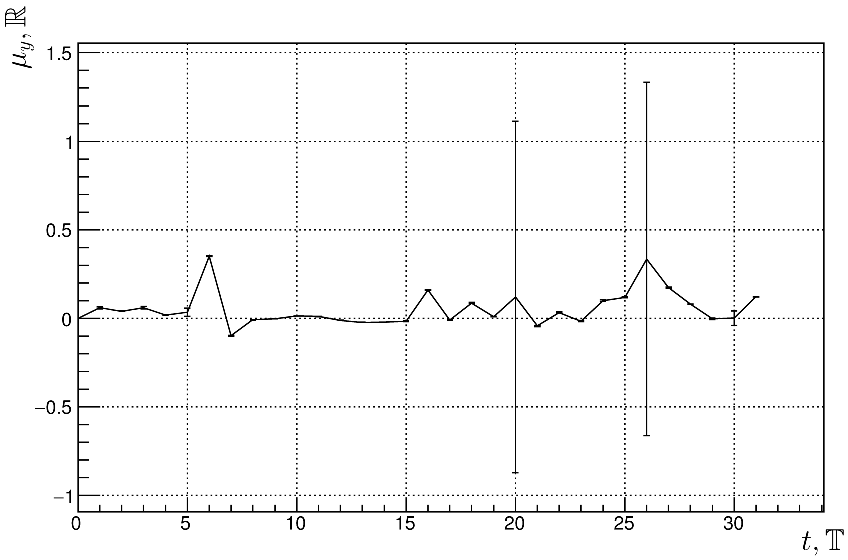

Figure 2: Mathematical expectation $\mu_y(t)$ of different combinations

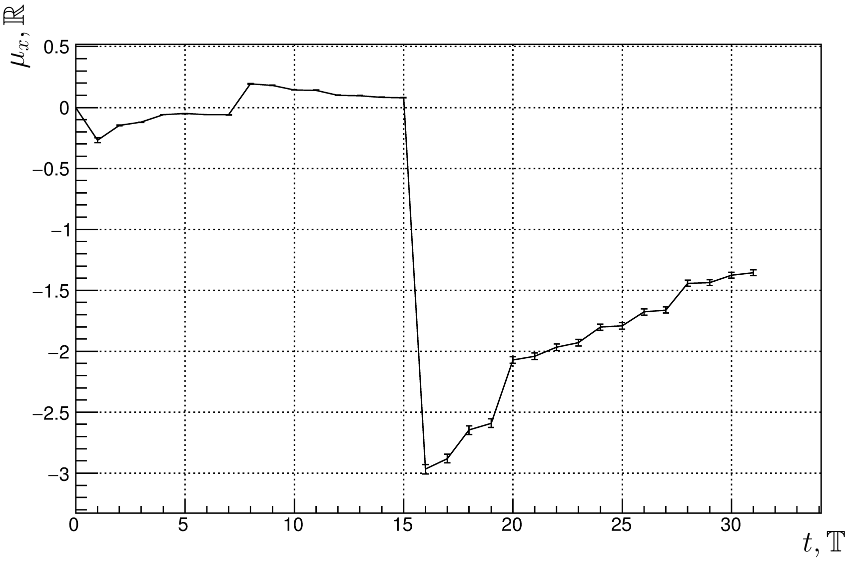

Figure 3: Mathematical expectation $\mu_z(t)$ of different combinations

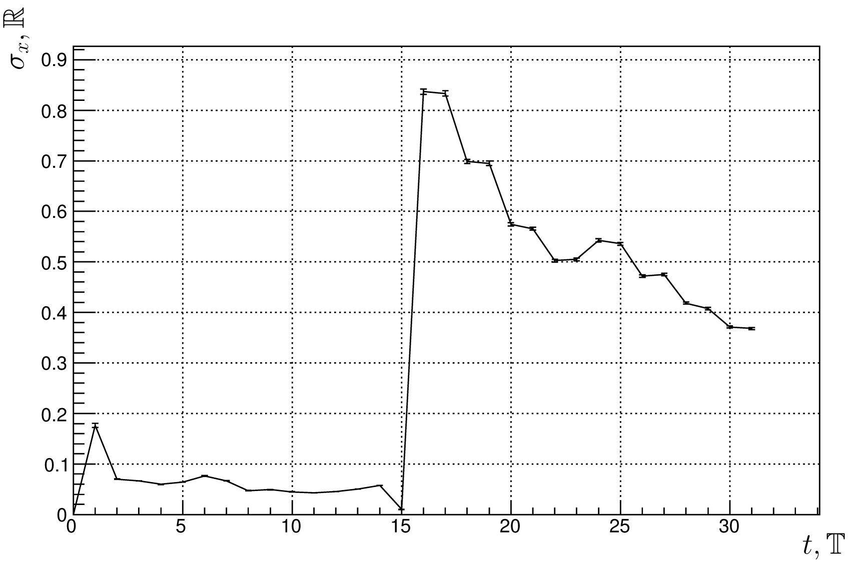

Figure 4: RMS $\sigma_x(t)$ of different combinations

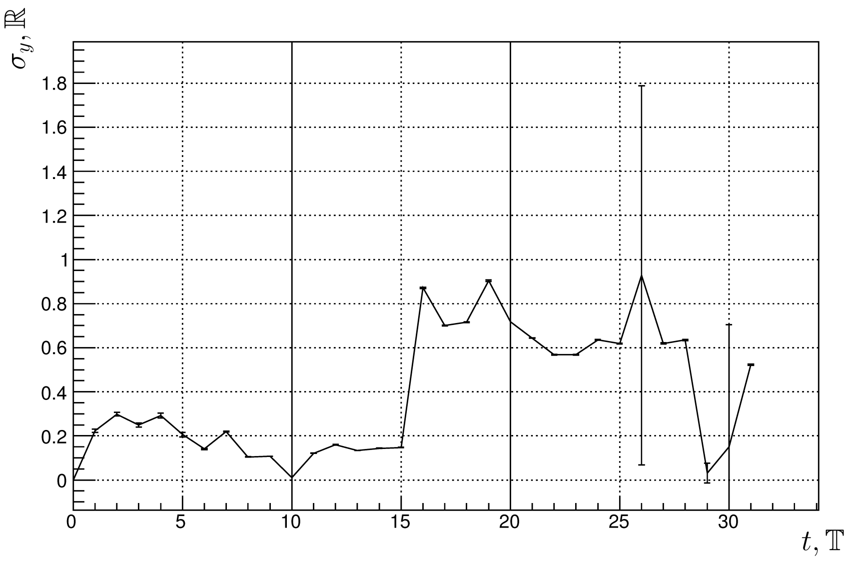

Figure 5: RMS $\sigma_y(t)$ of different combinations

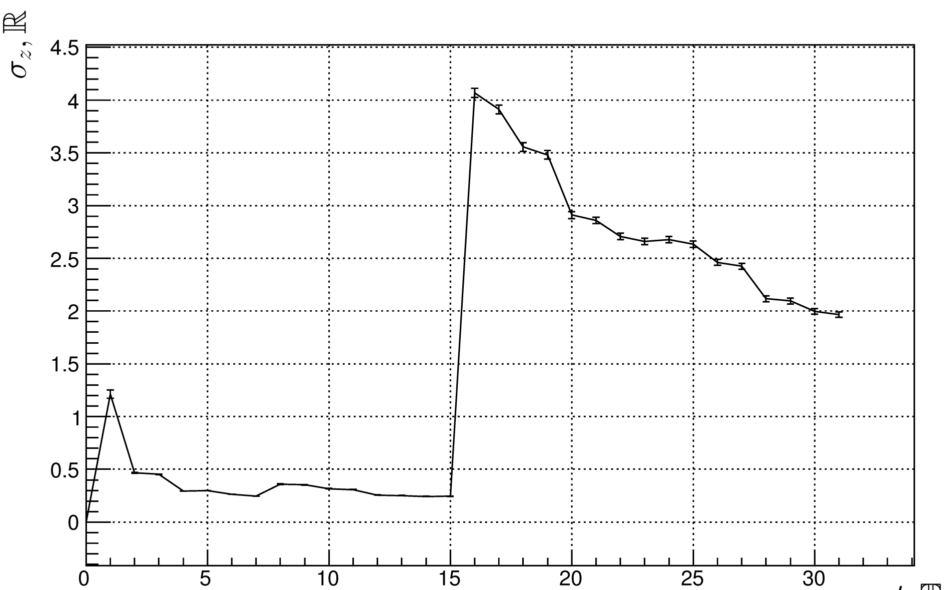

Figure 6: RMS $\sigma_z(t)$ of different combinations

Thus, it is possible to maintain that the best results shows the combination (15) without the shortest tracks.

However, only primary particles were used for investigation. To separate primary and secondary tracks, several charts were drawn, and collisions of Au were used. The fig.7 shows the correlation between the number of passed stations n and type of track (red points mark secondary tracks, blue points - primary tracks). Also this graph describes how often we register the different types of tracks (p).

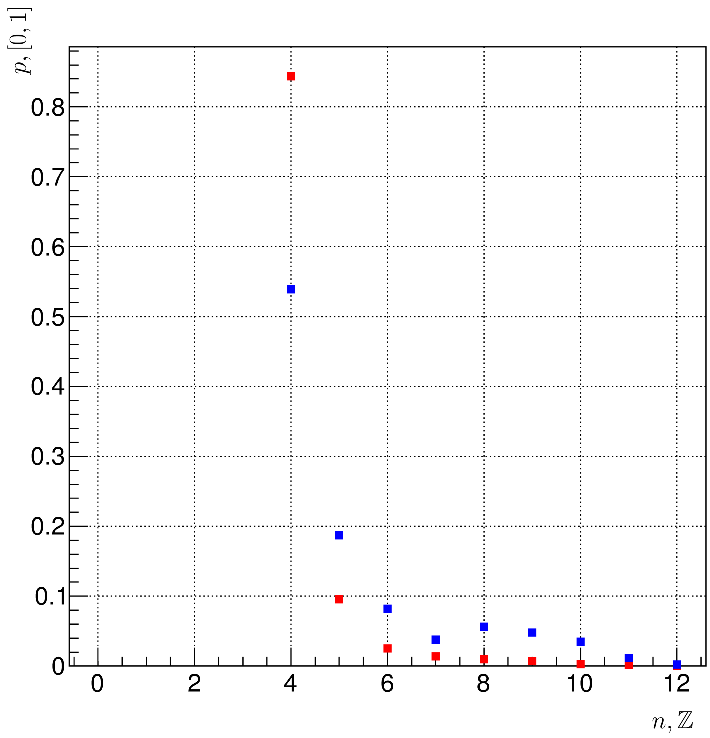

Figure 7: Correlation between the number of passed stations n and type of track

The fig.7 is quite similar with fig.8, however is the χ^2 criterion used instead of number of passed detectors.

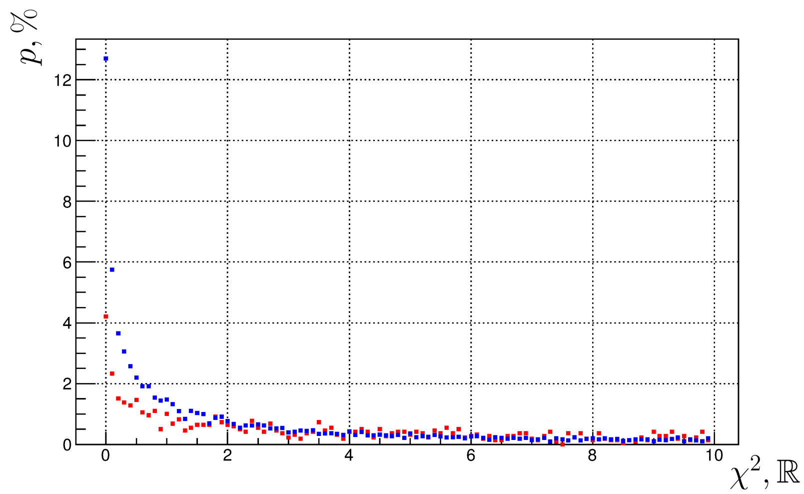

Figure 8: Distribution of particles $p(\chi^2)$ depending on χ^2

As we can see, approximately 0.9 of secondary tracks have 4 or 5 passed stations. It is also clear, that there are more primary tracks with χ^2 ¡ 0.1 than secondary. The percent of primary tracks, which has 4 passed stations and $\chi^2 < 0.1$, is 16 %, so it is possible to assume that both of criteria (number of points and χ^2) might be good for rejecting the secondary tracks, and, as a result, improving of primary vertex finding.

Algorithm of secondary vertex finding

Description of the algorithm

Not all of particles can be detected by GEMs. For instance, neutral particles pass via GEM without any signal in it. One of the such a difficult for discovery objects is Λ^0, which can fly away from primary vertex and decay between the target and GEM. Fortunately, there exists a decay mode $p^+\pi^-$ (64%), which could lead us to solution.

It should be noticed, that we have to compare all of particles with opposite charges in pairs due to lack of information about types of particles (the particle identification is still under development). So, we use the same algorithm for all of couples, and the stages of it are described below.

First of all, it is needed to find the first approximation of the distance of the closest approach (DCA). This step can be realized by "slicing" the axis z and computation the distance $|\vec{r}|$ between the points on the plane $z = const$, which were propagated by Kalman filter. As a result, we have the first approximations of DCA and the area of CA.

Next, we try to improve the accuracy of the measurements. To do that, we can approximate both of tracks in the area of CA by analytical functions and find the distance between the approximations. So, a S_3^2 splines were chosen for interpolation. However, we need to construct the spline in $\mathbb{R}^3$, so that the parametrisation is needed. Taking into consideration a geometry of BM@N and bijection between the point of track and z coordinate, we may select the $t = z$ parametrization. After that, it is possible to write a spline in a usual form:

$$\vec{S_i}(t) = \vec{\varphi_{0i}} + \vec{\varphi_{1i}}(t - t_i) + \vec{\varphi_{2i}}(t - t_i)^2 + \vec{\varphi_{3i}}(t - t_i)^3, \quad t \in \mathbb{R} \ \ \vec{\varphi} \in \mathbb{R}^3 \quad (6)$$

However, boundary conditions are also needed for construction. Fortunately, we can use the Kalman information about the track, particularly a direction vector of the track. So, we can write:

$$\vec{S_0}'(t_0) = \vec{T}(t_0), \tag{7}$$

$$\vec{S_{n-1}}'(t_n) = \vec{T}(t_n) \tag{8}$$

After that, the process of spline interpolation might be implemented by one of the few standard algorithms [8]. Although, there is only one modification for $\mathbb{R}^3$ case - here we have to do 3 identical iterations instead of 1 iteration in $\mathbb{R}^2$ case. We must do that because we have 3 equations

$$\vec{S_{ix}}(t), \quad \vec{S_{iy}}(t), \quad \vec{S_{iz}}(t) \tag{9}$$

instead of 1 equation in standard $\mathbb{R}^2$ case:

$$S_i(x) = y(x), \quad S_i \in \mathbb{R}, \quad x \in \mathbb{R} \tag{10}$$

Thus, we have an analytical approximations of the tracks in the CA area, consequently it is possible to find the distance between these curves. To do that, we write the system of non-linear equations. Two of equations are amenity and other two are orthogonality condition equations.

$$\vec{\varphi}(t) = \vec{\varphi_{0i}} + \vec{\varphi_{1i}}(t_1 - t_i) + \vec{\varphi_{2i}}(t_1 - t_i)^2 + \vec{\varphi_{3i}}(t_1 - t_i)^3 = \vec{p_0} \tag{11}$$

$$\vec{\psi}(t) = \vec{\psi_{0j}} + \vec{\psi_{1j}}(t_2 - t_j) + \vec{\psi_{2j}}(t_2 - t_j)^2 + \vec{\psi_{3j}}(t_1 - t_j)^3 = \vec{p_0} + \vec{r} \tag{12}$$

$$\frac{\partial \vec{\varphi}}{\partial t}\vec{r} = 0 \tag{13}$$

$$\frac{\partial \vec{\psi}}{\partial t}\vec{r} = 0 \tag{14}$$

Substitution of 11 into 12 gives 5 non-liner equations with 5 unknown variables $(\vec{r}, t_1, t_2)$. These equations may be solved by Newton method. As a result, the DCA and the point of CA are found.

However, sometimes finding of DCA using spline interpolation does not give good results. In this case it is possible to use parabola instead of spline. There are only 3 points needed to construct the parabola, and the systems of equations is quite similar with system for spline.

After the successful finding of DCA and the point of CA (we can say that it is a candidate for secondary vertex), it is needed to define a criteria, which will be used to identify the origin of the considered point of CA. These conditions, which were developed according to the topology of Λ^0 decay [9], will be discussed further.

The criterion of DCA is a maximum distance between the two tracks, that let us assume that considered point might be a secondary vertex. If the found DCA is longer then defined maximum distance, the couple of tracks will be rejected from the examination.

Another criterion is a "left edge" criterion. If the coordinate of the point of CA $z_{CA} < z_0$, where $z0$ is a pre-defined parameter, than we cannot consider this pair of tracks. It is useful to define $z_0 \approx 0$ to reject the primary particles.

The criteria discussed below are more specific. One of them is a "$\vec{p_1}\vec{p_2}$" criterion, which be used for two-body decays. To implement this criterion, we need to know the ratio $|\vec{p_1}|/|\vec{p_2}|$, where $\vec{p_1}$ and $\vec{p_2}$ are impulses of decay products. If considered ratio is too different from the defined one, we will reject this pair.

Another one specific criteria can be used for uncharged particles. A trajectory of this kind of particles before the decay is a straight line. So, we can calculate a $\vec{p_0} = \vec{p_1} + \vec{p_2}$ and find the distance between the primary vertex and the trajectory of the neutral particle. If the straight line is too far from the primary vertex, we can say that it is not the Λ^0 or K^0.

After the use of criteria, we decrease a number of selected candidates, however we have not identified the particles yet. To find a type of particle, an Armenteros-Podolanski plane can be used [10].

Analysis of efficiency

In this paragraph an efficiency of secondary vertex finding of Λ^0 will be discussed. First, let us have a look at Armenteros-Podolanski plane:

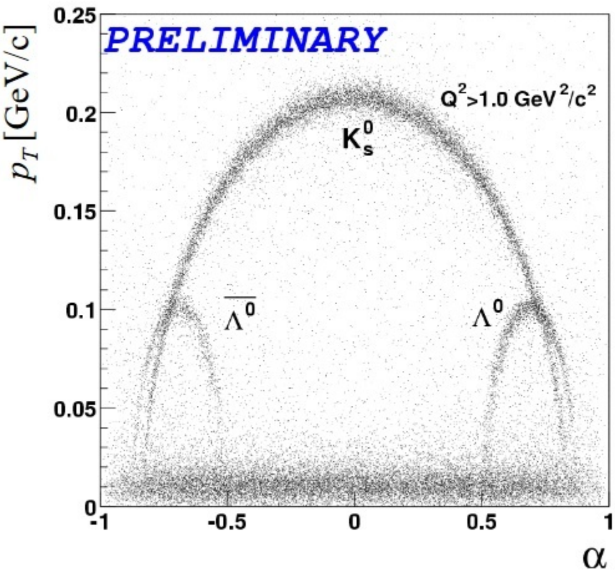

Figure 9: Armenteros-Podolanski plane (COMPASS experiment)

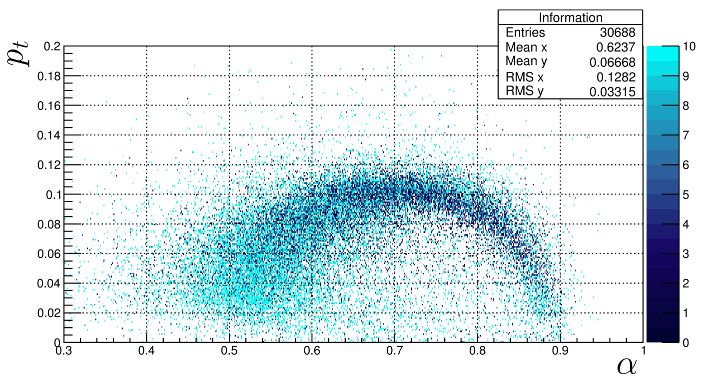

Figure 10: Armenteros-Podolanski plane (BM@N reconstruction, $2 \cdot 10^5$ events) [colourized online]

The fig.9,10, where $\alpha = \frac{p_L^+ - p_L^-}{p_L^+ - p_L^-}$ ($p_L^\pm = |\vec{p^\pm}|cos\varphi_\pm$), the colour marks the distance between Kalman propagation of z coordinate of the secondary vertex and Monte-Carlo value (all of distances ¿ 10 use the colour of grade 10) and $p_t^\pm = |\vec{p^\pm}|sin\varphi_\pm$, show the theoretical prediction (12) and the results of implementation (without a criteria for rejection) of algorithm for events, that contain only Λ^0. As we can see, the parabola on the second picture is quite diffuse, and the strongest blurring is located in the low α area. Also it is significant that the most of remote points has a bad defined secondary vertex. The analysis of the situation has showed that it usually had happen because of inaccuracy of a tracking.

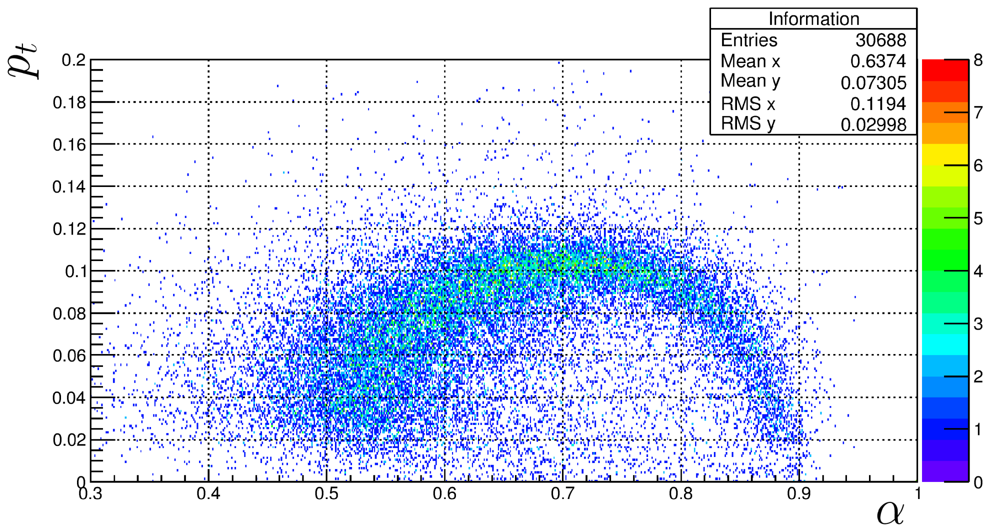

Figure 11: Armenteros-Podolanski plane, density of distribution [colourized online]

So, we may assume that the narrowing of the parabola is might be done by momentum reconstruction improvement. However, we have not discuss the cuts for events with the different type of secondary and primary particles yet. To do that, we will use a deuteron-carbon (LAQGSM generator) collisions.

First of all, the necessary for definition of criteria graphs were plotted. To do that, we use the events, that contain only Λ^0, again.

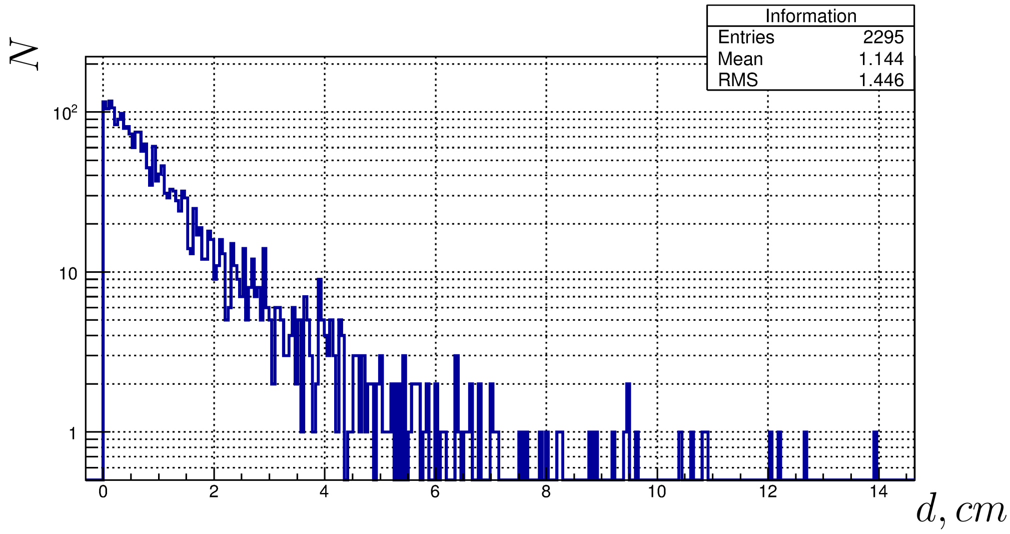

Figure 12: Distance of the closest approach

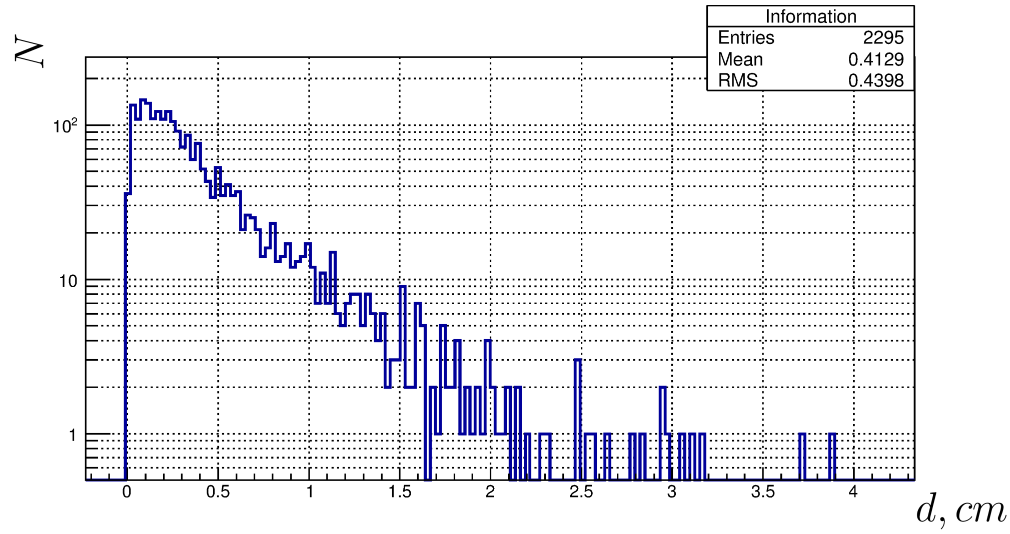

Figure 13: The distance between the $\vec{p_0}$ trajectory and the primary vertex

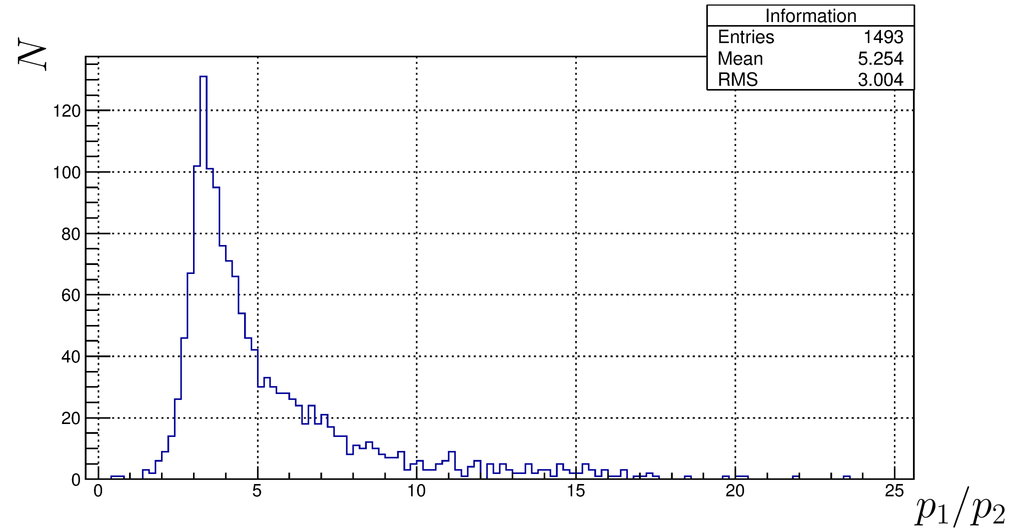

Figure 14: The ratio p_1/p_2

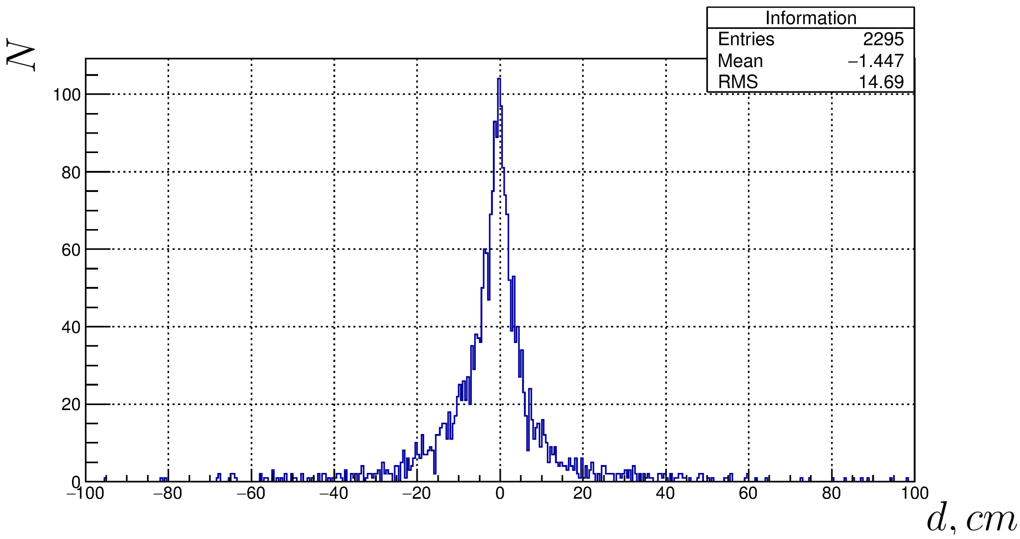

Figure 15: The distance d between reconstructed z_{0re} and MC z_{0mc}

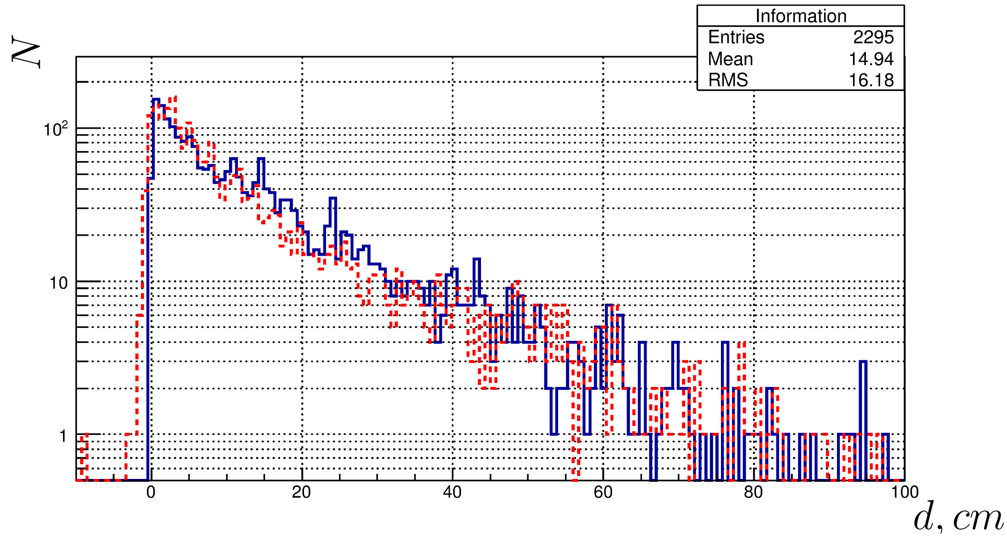

Figure 16: The distribution of z_0 of reconstructed and MC tracks [colourized online]

Taking all distributions into consideration, we found the best cuts. However, we also need to estimate an amount of particles, which could be detected and identified (for deuteron-carbon collisions, using all of cuts):

Total num. of detected Λ^0 1224	Num. of unique Λ^0 1028
Total num. of Λ^0, that could been iden. 259	Num. of unique Λ^0, that could been iden. 216
Total num. of simulated Λ^0 9676	Num. of unique rejected* Λ^0 399

Table 1: Table of characteristics

In the table [1] we had to define the notion "unique Λ^0", because some of the MC tracks of the products of Λ^0 decay were split during the reconstruction and, as a result, the one MC particle became a few reconstructed particles with their own tracks. Therefore, the number of unique particles is an amount of the true MC Λ^0, that has a detected products of the decay. The "*" means that these Λ^0 were rejected due to the inaccuracy of tracking. Particularly, there were the particles, which did not pass the "left edge = 0" criterion, however their MC z coordinate of the secondary vertex was ¿ 0. Actually, if they had passed the criterion, they would have a possibility to fall another cut, nevertheless we can claim that these Λ^0 are prospective.

After that, algorithm was implemented for deuteron-carbon collisions (with all of cuts).

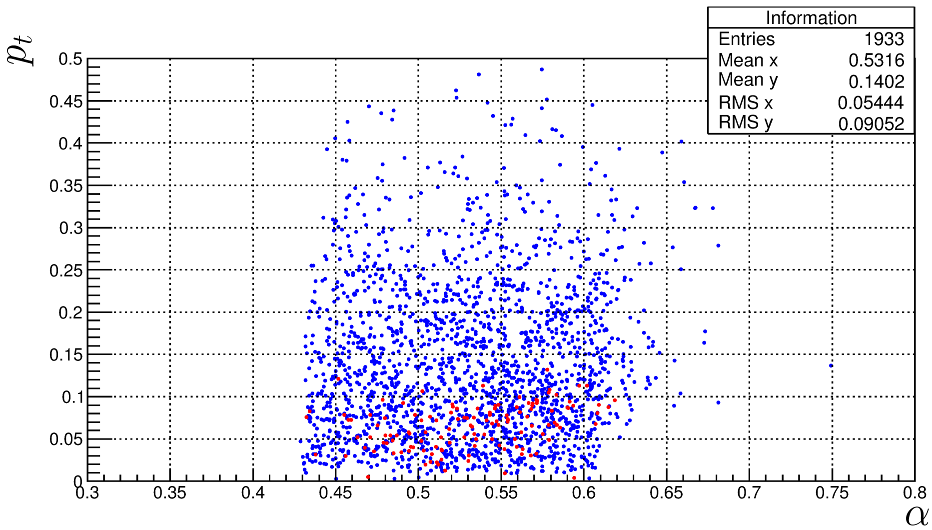

Figure 17: Armenteros-Podolanski plane for deuteron-carbon collisions ($10^5 events$

As a result, we can see, that "$\vec{p_1}\vec{p_2}$" criterion just bounds the α. Also, there are some noise (blue points) on the picture. To separate the red from blue points we used a parabolas and a straight line. Then, we assume that the particles in the area are Λ^0. The results are showed in the table [2]:

Total num. of particles	Total number of Λ^0
2063	130
Num. of particles in the area	Num. of Λ^0 in the area
538	86

Table 2: Table of characteristics

Finally, we can maintain, that approximately 21% of reconstructed Λ^0 could be identified, however only 14% of reconstructed Λ^0 are found after the separation of Λ^0 from the noise. Also it is significant that the noise does not rejected completely, and there are ratio $\Lambda^0/noise = 0.16$. The, it is possible to say that 14% of Λ^0 were identified and their secondary vertexes were found. However, we also found some noise. Nevertheless, the first realization of secondary vertex finding algorithm was done and there are new prospectives for improvement defined.

Conclusion

To sum up all of discussed issues, a few inferences might be done. One of them is about the algorithm of primary vertex finding. So, an efficiency of modified vertex finding were discovered, and now it is possible to use the best combination of tracks.

Another one conclusion is linked with the secondary vertex problem. There was an algorithm of finding implemented, and the efficiency of it was researched. As analysis showed, the 14% of reconstructed Λ^0 are found and the ratio $\Lambda^0/noise = 0.16$. However, a several ways of improvement of the situation were introduced.

It is also important to notice, that all of the results were implemented in BmnRoot (framework for simulation, reconstruction and analysing of data) software system [7]. Moreover, both of algorithms will be included to the systems as a tasks. Thus, they will provide a new opportunities for research at BM@N.

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